1. Find \( \lim_{x \to \infty} \left( \sqrt{x^2 + x} - x \right) \), if it exists. (2 points)

2. Find the constant \( \alpha \) such that the function \( f(x) = \begin{cases} x^2 + x + \alpha^2 & \text{if } x < 2 \\ 2\alpha x + 2 & \text{if } x \geq 2 \end{cases} \) is continuous. (2 points)

3. If \( xy + y^2 = 2 \), find \( y'' \) at \((1,1)\). (2 points)
4. Given the cost of producing $q$ units of a certain product is $C = q^2 - 5q + 3$, find the value of $q$ at which the marginal cost is 1. (2 points)

5. If at $x = 1$ \( \frac{d}{dx} \left[ x^k - k^x \right] = 0 \), find the positive constant $k$. (2 points)

6. Given $f(x, y) = y^2 e^x + \ln(xy)$, find $f_{xy}(1,1)$ (2 points)

7. Examine $f(x, y) = x^2 + 2y^2 - 2xy - 4y + 3$ for relative extrema. (3 points)
8. Find the area bounded by \( y = x^2 \), \( y = 1 \), and \( y = 4 \). (2 points)

9. Integration
   
   a. \( \int \frac{\cos x}{\sin x + 1} \, dx \) (2 points)

   b. \( \int x^3 \, dx \) (2 points)

   c. \( \int \csc^2 x \cot^3 x \, dx \) (2 points)
d. \[ \int_0^1 x^2 \sqrt[3]{7x^3 + 1} \, dx \] (2 points)

e. \[ \int_{10}^{10} \sin(\sin(x^2)) \, dx \] (2 points)

f. If \( f''(x) = 6x + 2 \) and \( f'(-1) = 5 \), find \( f(1) - f(-1) \). (2 points)

g. If \( \int_1^4 f(x) \, dx = 6 \), \( \int_2^4 f(x) \, dx = 4 \), and \( \int_1^3 f(x) \, dx = 5 \), find \( \int_2^3 f(x) \, dx \). (2 points)
10. Consider the function \( f(x) = x^{\frac{1}{3}}(x - 8) \) (6 points)

a. Find the first and second derivatives.

b. Find the \( x \) – and \( y \) – intercepts.

c. Find all critical numbers, if any exist.

d. Find the relative extrema, if any exist, and where \( f \) is increasing or decreasing.

e. Find inflection points, if any exist, and where \( f \) is concave up or down.
f. Sketch the graph of $f(x)$ clearly indicating all important points and concavity.

11. Use the definition of the derivative to find $f'(x)$ given that $f(x) = x^2$ (3 points)