1. Show complete and neat work for full credit.

2. This exam contains (9) pages.
1. Identify and give a rough sketch of the surfaces: (10 points)

   (a) \[4x^2 - y^2 + 16(z - 2)^2 = 1.\]

   (b) \[z = \sqrt{1 + x^2 + y^2}.\]
2. Find equation of the surface \( z = x^2 - y^2 \): (5 points)

(a) in cylindrical coordinates

(b) in spherical coordinates.
3. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$. (5 points)

(a) Find $\lim_{(x, y) \to (0, 0)} f(x, y)$.

(b) Is $f(x, y)$ differentiable at $(0, 0)$? Give reasons.
4. For \( z = \frac{x}{x + y} \), show that (10 points)

(a) \( z_{xy} = z_{yx} \)

(ii) \( z_{xx} + z_{yy} \neq 0. \) (10 points)
5. Let \( y = w \tan^{-1}(uv) \) where

\[
\begin{align*}
  u &= r + s \\
  v &= s + t \\
  w &= t + r
\end{align*}
\]

Use chain rule to find \( \frac{\partial y}{\partial s} \) when \( r = 1 \), \( s = 0 \) and \( t = 1 \). (5 points)
6. Find the directional derivative of \( f(x, y) = \sqrt{xy} \) at \( P(2, 8) \) in the direction of \( Q(5, 4) \).
(5 points)
7. Find points on the surface \( x^2 + y^2 + z^2 = 1 \) at which normal line is parallel to the line through the points \( P(1, -2, 1) \) and \( Q(4, 0, -1) \). \hspace{1cm} (10 points)
8. Examine the function \( f(x, y) = xy - x^3 - y^2 \) for local extrema and saddle points. (10 points)