King Fahd University of Petroleum and Minerals
Department of Math & Stat

Math 201 (Calculus III) (071)
Major Examination II (Sec.#: 2, 5, 14)
December 2, 2007

Time: 90 Minutes **Marks: ________/60**

Name: ______________________________________

ID #: ________________________ Serial #: __________ Sec.#: ____________

---

1. Show complete and neat work for full credit.
2. This exam contains (9) pages.

1. Identify and give a rough sketch of the surfaces: (10 points)

   (a) $4x^2 - y^2 + 16(z-2)^2 = 1$.

   (b) $z = \sqrt{1 + x^2 + y^2}$.

2. Find equation of the surface $z = x^2 - y^2$: (5 points)

   (a) in cylindrical coordinates

   (b) in spherical coordinates.

3. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$. (5 points)

   (a) Find $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$.

   (b) Is $f(x, y)$ differentiable at $(0, 0)$? Give reasons.

4. For $z = \dfrac{x}{x+y}$, show that (10 points)

(a) $z_{xy} = z_{yx}$

(ii) $z_{xx} + z_{yy} \neq 0$. (10 points)

5. Let $y = w \tan^{-1}(uv)$ where

$$\begin{aligned} u &= r + s \\ v &= s + t \\ w &= t + r \end{aligned}$$

Use chain rule to find $\dfrac{\partial y}{\partial s}$ when $r = 1,\ s = 0$ and $t = 1$. (5 points)

6. Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$.
(5 points)

7. Find points on the surface $x^2 + y^2 + z^2 = 1$ at which normal line is parallel to the line through the points $P(1, -2, 1)$ and $Q(4, 0, -1)$. (10 points)

8. Examine the function $f(x, y) = xy - x^3 - y^2$ for local extrema and saddle points. (10 points)