King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
**Math 201 (Sections 2, 5, 14)**
**Final Examination**
**Semester I, 2007–2008(071)**
**January 22, 2008**
**Time: 07:00 p.m. – 10:00 p.m.**

---

Name: ______________________ Section #: __________

ID #: ______________ Serial #: __________

---

| Instructor | Location |
|---|---|
| Dr. Abdul Rahim Khan | Gymnasium |

**Instructions**:

1. Do not use programmable calculators. Use of ordinary calculator is allowed.
2. Show all your work. Less credit will be given for answers not supported by proper work.
3. This exam consists of 13 pages.
4. Do not forget to write your NAME, ID, Section # and Serial # in the space provided above.

| Question # | Grade/Points |
|---|---|
| 1 | ________/18 |
| 2 | ________/18 |
| 3 | ________/18 |
| 4 | ________/18 |
| 5 | ________/18 |
| 6 | ________/20 |
| Total: | ________/110 |

1. (a) Test the polar equation $r = \cos 2\theta$ for symmetry and draw its graph by giving all necessary details. (9 points)

(b) Calculate area of the region that is common to the circles $r = 4\cos\theta$ and $r = 4\sin\theta$. (9 points)

2. (a) Find for the lines:

$$L_1: \quad x = 4t, y = 1 - 2t, z = 2 + 2t$$
$$L_2: \quad x = 1 + t, y = 1 - t, z = -1 + 4t$$

(i) point of intersection (5 points)

(ii) angle between $L_1$ and $L_2$. (4 points)

(b) Sketch the surfaces

$$S_1: \; z = x^2 + y^2$$
$$S_2: \; z = 6 - x^2 - y^2$$

Also describe and sketch curve of intersection between $S_1$ and $S_2$.

(9 points)

3. (a) Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $(x_0, y_0, z_0)$ is $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$. (9 points)

(b) Find a unit vector in the direction in which $f(x, y, z) = 4e^{xy} \sin z$ decreases most rapidly at $P\left(1, 0, \dfrac{\pi}{4}\right)$ and find the rate of change of $f$ at $P$ in that direction. (9 points)

4. (a) Find maximum and minimum values of $f(x, y) = xy - x^3y^2$ over the region $R = \{(x, y) : 0 \leq x \leq 1, \quad 0 \leq y \leq 1\}$. (9 points)

(b) Use the Lagrange multiplier method to find points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point $(1, 2, 2)$. (9 points)

5. (a) Sketch the region $R$ bounded by the graphs of the equations $y = x, y = 3x$ and $x + y = 4$. Use double integrals to find area of $R$. (9 points)

(b) Use polar coordinates to:

(i) Evaluate $\displaystyle\int_1^2 \int_0^x \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx.$ (5 points)

(ii) Set up an iterated integral for the volume of solid bounded above by the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$. (4 points)

6. (a) Find volume of the solid enclosed between the paraboloids $z = 5x^2 + 5y^2$ and $z = 6 - 7x^2 - y^2$. (10 points)

(b) Use spherical coordinates to find volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 4$ and the $xy$-plane. (10 points)