King Fahd University of Petroleum & Minerals  
Department of Mathematics and Statistics  
2007-2008 (071)  
Calculus III (MATH 201)  
Final Exam

<table>
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<th>Student Name:</th>
<th>Section #:</th>
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INSTRUCTIONS

1. For Questions 1 to 6 show your work.  
2. For Questions 7 to 12 circle the correct answer in the table below.  
3. Write clearly and legibly. Marks may be deducted for messy work.

<table>
<thead>
<tr>
<th>Question</th>
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<td>TOTAL</td>
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1. For the function \( f(x, y) = 4x^3 + y^3 - 6x^2 - 6y^2 + 5 \), find
(a) the critical points;
(b) the local maximum and minimum values and saddle point(s).
2. Find symmetric equations for the line $L$ through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t, y = 1 - t, z = 2t$. 
3. By reversing the order of integration, evaluate \[ \int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3} \, dx \, dy. \]
4. Set up a triple integral for the volume of the solid bounded by the surfaces \( y = 2 - z^2 \), \( y = z^2 \), \( x + z = 4 \), \( x = 0 \). Evaluate the integral.
5. Use cylindrical coordinates to evaluate \( \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{(x^2+y^2)^2}^{16} x^2 \, dz \, dy \, dx \).
6. Find the volume and the centroid of the region bounded above by the sphere $\rho = 4 \cos \phi$ and below by the cone $\phi = \pi/6$. 
7. Using the linear approximation of the function \( f(x, y) = \sqrt{2x^3 + y^2} \) at \((2, 3)\), we find that \( f(2.03, 2.97) \approx \)

(a) 5.01  
(b) 5.02  
(c) 5.03  
(d) 5.04  
(e) 5.05

8. If \( x e^{yz} - 2ye^{xz} + 3ze^{xy} = 0 \), then \( 3 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \) at the point \((0, 0, 0)\) is equal to

(a) \(-1/3\)  
(b) \(1/3\)  
(c) \(-4/3\)  
(d) \(4/3\)  
(e) \(-2/3\)

9. If Lagrange multipliers are used to find the maximum \( M \) of \( f(x, y) = x^2 + y^2 \) subject to the constraint \( g(x, y) = x^4 + y^4 = 1 \), then \( M \) is equal to

(a) \(3\sqrt{2}\)  
(b) \(2\sqrt{3}\)  
(c) \(2\sqrt{2}\)  
(d) \(\sqrt{3}\)  
(e) \(\sqrt{2}\)
10. The distance between the skew lines $L_1 : x = t, y = t, z = t$ and $L_2 : x = -1 + t, y = 3t, z = 2t$ is:

(a) $2/\sqrt{6}$
(b) $1/\sqrt{3}$
(c) $2/\sqrt{3}$
(d) $1/\sqrt{2}$
(e) $1/\sqrt{6}$

11. The absolute maximum of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disk $D = \{(x, y) \mid x^2 + y^2 \leq 4\}$ is

(a) $2/e^4$
(b) $2/e^2$
(c) $4/e^2$
(d) $2/e$
(e) $4/e$

12. The length of the parametric curve: $x = 5 + 6t, y = e^{3t} + e^{-3t}$ $(0 \leq t \leq \ln 2)$ is equal to

(a) $65/8$
(b) $63/8$
(c) $61/8$
(d) $59/8$
(e) $57/8$