1. Write clearly.

2. Show all your steps.

3. No credit will be given to wrong steps.

4. Do not do messy work.

5. Calculators and mobile phones are NOT allowed in this exam.
1. (6-point) Determine the form of the particular solution of the DE

$$\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 4y = te^t + \cos(2t) + t + 2$$

(Do not find the values for the constants)

2. (5-points) Find the regular and irregular singular points of the DE

$$x^2(x - 2)^2 y'' + 3xy' + (x^2 - 4)y = 0$$
3. (3-points) Suppose that \( x = 0 \) is a regular singular point of a second order DE with \( r_1 = \frac{3}{2} \) and \( r_2 = 0 \) be the indicial roots of the singularity. Find two linearly independent series solution of the DE.

4. (3-points) Given that: \( y = c_1e^x \cos x + c_2e^x \sin x \), is a general solution of the DE \( y'' - 2y' + 2y = 0 \).

Find a solution of (1) that satisfies the boundary following conditions \( y(0) = 1 \), \( y'(\pi) = 0 \).
5. (5-points) The functions: \(y_1 = 1, y_2 = x, y_3 = \cos x, y_4 = \sin x\), are solutions of the DE:

\[ y^{(4)} + y'' = 0; \quad \text{on} \quad (-\infty, \infty). \]

Show that \(y_1, y_2, y_3, y_4\) form a fundamental set of solutions of the DE (2). Find the general solution.
6. (12-points) Solve the following DE by using the variation of parameters method

\[ y'' - 2y' + y = \frac{e^x}{1 + x^2} \]

**Hint:** \[ \int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C \]
7. (6-points) Solve the Cauchy-Euler equation

\[ x^2 y'' - 2xy' - 10y = 0 \]
8. (10-points) Use the power series to solve the following differential equation

\[
\frac{d^2y}{dx^2} + x \frac{dy}{dx} + 2y = 0
\]
9. (5-points(Bonus)) The function: \( y_1 = x \sin(\ln(x)) \), is a solution of

\[
x^2 y'' - xy' + 2y = 0 \quad I = (0, \infty).
\]

Find a second solution \( y_2 \) of the given DE on \( I \).

**Hint:** \( \int \frac{dt}{\sin^2 t} = -\cot t + C \).