King Fahd University Of Petroleum and Minerals

College of Sciences

Mathematics and Statistics Department

Math 202

Major Exam II

Section 7

Name:................................ ID#:............... Serial #:...........

NO CALCULATOR IS ALLOWED IN THE EXAM

SHOW ALL NECESSARY WORK

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Instructor: H. Al-Attas
1. (9 points) Given a two-parameter family \( y = c_1 x^2 + c_2 x^4 + 3 \) is a solution of the differential equation \( x^2 y'' - 5xy' + 8y = 24 \). Determine whether a member of the family can be found that satisfies the boundary conditions:

   (a) \( y(-1) = 0, \ y(1) = 4 \)

   (b) \( y(-1) = 0, \ y'(1) = 4 \)

2. (7 points) Verify that the functions \( \sin (\ln x) \) and \( \cos (\ln x) \) form a fundamental set of solutions of the differential equation \( x^2 y'' + xy' + y = 0 \) on the interval \((0, \infty)\). Find the general solution.
3. (6 points)

(a) By inspection find a particular solution of $y'' + 2y = 10$.

(b) By inspection find a particular solution of $y'' + 2y = -4x$.

(c) Find a particular solution of $y'' + 2y = -4x + 10$.

(d) Find a particular solution of $y'' + 2y = 8x + 5$.

4. Solve the following differential equations

(a) (8 points) $x^2 y'' - xy' + 2y = 0$ given that $y = x \sin (\ln x)$ is a solution. (Use Reduction of Order Method)
(b) (9 points) $y''' + 12y'' + 36y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -7$. 
(c) (10 points) \( y^{(4)} - 4y' = 5x^2 - e^{2x} \).
(d) (12 points) $y'' + 3y' + 2y = \sin e^x$. 
(e) (13 points) $x^2y'' - 4xy' + 6y = \ln x^2$. (Use the Substitution $x = e^t$)
(f) (6 points) $25x^2y'' + 25xy' + y = 0$.

(g) (Bonus) Find a linear second-order differential equation with constant coefficients for which $y_1 = 1$ and $y_2 = e^{-x}$ are solutions of the associated homogeneous equation and $y_p = \frac{1}{2}x^2 - x$ is a particular solution of the nonhomogeneous equation.