Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Q 8	Total mark

Table 1: Table of Marks

King Fahd University of Petroleum and Minerals

First Major Exam for Math 260

Semester 1, Academic year 2007-2008

Time allowed 1 hour and 30 minutes

Full Name: ....

ID Number: ....

Section: ....

Note The following things are prohibited

- Using an advanced calculator
- Having the mobile phone on
- Talking to each other
- Cheating

Q 1 (4 Points) Consider the following nonlinear DE:

$$y' + 2xy^2 = 0 \qquad (1)$$

a) Verify that the function $y_1 = \frac{1}{1+x^2}$ is a solution of the DE (1)
b) Determine by inspection one other solution of (1)

Q 2 (4 Points) Consider the separable DE

$$(y-2)\frac{dy}{dx} = 2y^2\cos^2(x) + 3\sin^2(x) - 3\cos^2(x) - y^2 \tag{2}$$

a) Describe how the DE (2) can be solved by the method of separation of variable.
(In other ward, state the steps for solving the DE (2) without finding its solution.)
b) Find the singular solution of the DE (2).

Q 3 (5 Points) Solve the linear DE:

$$x\,\frac{dy}{dx} + x\,y\ln(x) = \frac{\mathrm{e}^{\left(1-\frac{x}{3}\right)\ln(x^3)}\mathrm{e}^{x}}{1+x^2}$$

Q 4 (5 Points) Consider the DE:

$$2xy\,\mathrm{e}^{x^2y}dx + x^2(\mathrm{e}^{x^2y} + x^{-2}\mathrm{e}^{-2y})dy = 0 \tag{3}$$

a) Prove that the DE (3) is exact.
b) Solve the DE (3).

Q 5 (6 Points) Use an appropriate substitution to solve the second order DE:

$$(4 - x^2)y'' - 2xy' = 0$$

on the interval $(-2, 2)$. (Hint: You may assume that y' is positive if helpful.)

Q 6 (7 Points) a) Verify that the DE

$$\frac{dy}{dx} = \frac{y(x+y)}{(x-y)(x+2y)}$$

is homogeneous

b) Solve the IVP:

$$\frac{dy}{dx} = \frac{y(x+y)}{(x-y)(x+2y)} \quad \text{and} \quad y(1) = 1.$$

on the interval $I = (a, b)$ where $0 < a < 1 < b$. (Note: You do not have to find a and b)

Q 7 (5 Points) Let α be a real constant. Consider the linear system:

$$\begin{cases} 2x - 3y - 2z & = 5 \\ 10x - 14y - 8z & = 25 \\ -4x + 9y + (\alpha^2 + 9)z & = \alpha - 11 \end{cases}$$

Write the augmented matrix form of the given system. Then use the Gauss elimination to find the values of α for which the system has

a) no solution

b) a unique solution

c) infinitely many solutions

Q 8 (4 Points) Use Gauss-Jordan elimination to find, if any, the set of all nontrivial solution of the homogeneous linear system

$$\begin{cases} 3x - 6y - 3z + 7t = 0 \\ -2x + 4y - 5z - 7t = 0 \\ x - 2y + z + 3t = 0 \end{cases}$$