Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Total mark

Table 1: Table of Marks

King Fahd University of Petroleum and Minerals

Second Major Exam for Math 260

Semester 1, Academic year 2007-2008

Time allowed 1 hour and 20 minutes

Full Name: ....

ID Number: ....

Section: ....

Note The following things are prohibited

- Using an advanced calculator
- Having the mobile phone on
- Talking to each other
- Cheating

Q 1 (5 Points) a) Determine whether or not the vectors $u = (3, -1, 2)$, $v = (5, 4, -6)$ and $w = (8, 3, -4)$ form a basis for R^3 (Justifications of our answer is required)

b) Determine whether the vectors $u = (1, 1, 0)$, $v = (0, 1, 0)$, $w = (0, 0, 1)$ and $t = (2, -2, 3)$ are linearly dependent or not.

Solution

Q 2 (3 Points) Let A be a square and invertible matrix with $A^T = A^{-1}$. Prove that the determinant of A is either 1 or -1.

Solution

Q3 (6 Points) Find a 3×3 matrix X such that $AX = B$ where

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & -2 \\ 1 & 7 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Solution

Q 4 (6 Points) Let

$$V = \{(x, y, z) \in R^3 \ \text{ such that } \ |x| = |y|\}$$

$$W = \{(x, y, z) \in R^3 \ \text{ such that } \ x + y + z = 0\}.$$

Show that V is not a subspace of R^3 and W is a subspace of R^3.

Solution

Q5 (3 Points) Let v_1 and v_2 be two linearly independent vectors. Apply the definition of linear independence to show that the vectors $u_1 = v_1 + v_2$ and $u_2 = v_1 - v_2$ are linearly independent.

Solution

Q6 (5 Points) Find a basis and the dimension for the solution space of the homogeneous system

$$\begin{cases} x_1 + 3x_2 + 4x_3 + 5x_4 = 0 \\ 2x_1 + 6x_2 + 9x_3 + 5x_4 = 0 \end{cases}$$

Solution

Q7 (8 Points) a)Verify that $y_1 = e^x$, $y_2 = e^x \cos x$ and $y_3 = e^x \sin x$ are solutions of the DE

$$y''' - 3y'' + 4y' - 2y = 0. \tag{1}$$

b) Use part (a) to find the general solution of the DE (1)

c) Solve the IVP

$$y''' - 3y'' + 4y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0.$$

Solution