Problem 1 (18 points): Define each of the following

(a) Nonseparable graph

(b) Complete symmetric digraph

(c) Isomorphic graphs

(d) Eccentricity

(e) Radius of a graph

(f) The Matrix-Tree Theorem.
Problem 2 (20 points): Consider the graph $G$ in the figure.

a) Find the
   (i) number of bridges = ____. List them: {}
   (ii) number of cut vertices = ____. List them: {}
   (iii) number of blocks = ____ . Draw them

b) Find a spanning tree of $G$.

c) Find
   (i) eccentricity $e(v_i) =$
   (ii) rad $(G) =$
   (iii) diam$(G) =$
   (iv) center $Z(G) =$
Problem 3 (10 points): Consider the join $G = G_1 + G_2$ of two graphs $G_1$ and $G_2$.
Which of the following statements is true? If so sketch a proof; and if not give a counter example.

a) If $G_1$ and $G_2$ are regular, then $G_1 + G_2$ is regular.

b) If $G_1$ and $G_2$ are bipartite, then $G_1 + G_2$ is bipartite.

c) If $G_1$ and $G_2$ are complete, then $G_1 + G_2$ is complete.

Problem 4 (10 points):
1) Draw a caterpillar $T$ of order 7 and find the Prufer sequence of $T$.

2) Draw a tree $T$ whose Prufer sequence is $(4 4 4 5 5 5)$. 
Problem 5 (15 points): Consider the graph $G = k_4 - e$, where $e$ is any edge of $G$. Label the vertices $v_1, v_2, v_3, v_4$.

a) Find the adjacency matrix $A = [a_{ij}]$ of $G$.

b) What is the graph theoretic meaning of the entry $a_{ij}^{(3)}$ of $A^3$?

c) Find $\sum_{i=1}^{4} a_{ii}^{(3)}$; (i.e., find the sum of the entries in the main diagonal of the matrix $A^3$) without using matrix multiplication.

d) Use matrices to find the number of labeled spanning trees of $G$. 
Problem 6 (28 points): DO ONLY FOUR PROBLEMS
In this problem $G$ is a graph of order $n \geq 2$ and size $m$. Either prove or disprove each of the following statements. If a statement is true sketch the proof and if it is false, give a counter example.
1) If $\delta(v) \geq n/2$ for each vertex $v$, then $G$ is connected.

2) If $v$ is a cut vertex of $G$, then $v$ is a cut vertex of the complement $\overline{G}$.

3) If $G$ is an acyclic graph with $m = n-1$, then $G$ is a tree.
4) If $G$ has only two vertices of odd degree, then there is a path in $G$ joining them.

5) The sequence $(1, 1, 3, 4, 5, 6, 7, 7)$ is a degree sequence of a graph. (Either construct such a graph or disprove the claim.)