**King Fahd University of Petroleum and Minerals**
**Department of Mathematics and Statistics**
**Math 425 - Graph Theory**
**Semester – 071**

**Exam I** **Dr. M. Z. Abu-Sbeih** **October 24, 2007**

Student No.: __________. Name: ___________________________________

*Show all your work. No credits for answers without justification.*
*Write neatly and eligibly. You may loose points for messy work.*

**Problem 1 (18 points):** Define each of the following

(a) Nonseparable graph

(b) Complete symmetric digraph

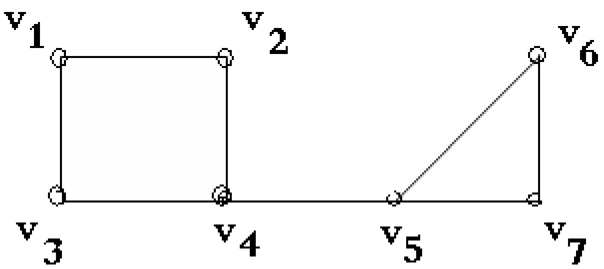
(c) Isomorphic graphs

(d) Eccentricity

(e) Radius of a graph

(f) The Matrix-Tree Theorem.

**Problem 2 (20 points):** Consider the graph *G* in the figure.




a) Find the (i) number of bridges = _____. List them: { }

(ii) number of cut vertices = ____. List them: { }

(iii) number of blocks = ____ . Draw them

b) Find a spanning tree of *G*.

c) Find (i) eccentricity $e(v_1) =$

(ii) rad ( *G* ) =

(iii) diam( *G* ) =

(iv) center Z( *G* ) =

**Problem 3 (10 points):** Consider the *join* $G = G_1 + G_2$ of two graphs $G_1$ and $G_2$. Which of the following statements is true? If so sketch a proof; and if not give a counter example.

a) If $G_1$ and $G_2$ are regular, then $G_1 + G_2$ is regular.

b) If $G_1$ and $G_2$ are bipartite, then $G_1 + G_2$ is bipartite.

c) If $G_1$ and $G_2$ are complete, then $G_1 + G_2$ is complete.

**Problem 4 (10 points):**

1) Draw a caterpillar *T* of order 7 and find the Prufer sequence of *T*.

2) Draw a tree $T$ whose Prufer sequence is ( 4 4 4 5 5 5).

**Problem 5 (15 points):** Consider the graph $G = k_4 - e$, where e is any edge of *G*. Label the vertices $v_1, v_2, v_3, v_4$.

a) Find the adjacency matrix $A = [a_{hj}]$ of *G*.

b) What is the graph theoretic meaning of the entry $a_{ij}^{(3)}$ of $A^3$ ?

c) Find $\sum_{i=1}^{4} a_{ii}^{(3)}$ ; (i.e., find the sum of the entries in the main diagonal of the matrix $A^3$) without using matrix multiplication.

d) Use matrices to find the number of labeled spanning trees of *G*.

**<u>Problem 6 (28 points):</u> DO ONLY FOUR PROBLEMS**

In this problem $G$ is a graph of order $n \geq 2$ and size $m$. Either prove or disprove each of the following statements. If a statement is true sketch the proof and if it is false, give a counter example.

1) If $\delta(v) \geq n/2$ for each vertex $v$, then G is connected.

2) If $v$ is a cut vertex of $G$, then $v$ is a cut vertex of the complement $\overline{G}$.

3) If $G$ is an acyclic graph with $m = n-1$, then $G$ is a tree.

4) If *G* has only two vertices of odd degree, then there is a path in *G* joining them.

5) The sequence (1, 1, 3, 4, 5, 6, 7, 7) is a degree sequence of a graph. (Either construct such a graph or disprove the claim.)