King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 425 - Graph Theory
Semester – 071

Exam II **Dr. M. Z. Abu-Sbeih** **December 5, 2007**

Student No.: _____. Name: _____

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (20 points):

1. Define each of the following

   (a) Connectivity of a graph
   (b) Strongly connected digraph
   (c) Hamiltonian connected graph
   (d) Pancyclic graph
   (e) Closure of a graph G

2. State each theorem:
   (a) Menger's Theorem:
   (b) The max-flow min-cut theorem for a network.

Problem 2 (20 points):

1. Give three conditions each of which implies that a graph G is
   (a) Eulerian.
   (b) Hamiltonian.

2. Consider the complete bipartite graph $G = K_{m,n}$.
   (a) For what values of n and m the graph G is Eulerian
   (b) For what values of n and m the graph G is Hamiltonian.
   (c) Find $\kappa(G)$ and $\kappa_1(G)$

Problem 3 (20 points):

Which of the following statements is true? If so sketch a proof; and if not give a counter example.

1. If a graph G is Eulerian then it has no bridges.
2. If a graph G is Eulerian and Hamiltonian then $|E(G)| = |V(G)|$.
3. A digraph D is strongly connected iff it is Eulerian.
4. If a digraph D is strongly connected then each pair of distinct vertices lie on a common cycle.
5. In a connected graph, each u-v walk contains a u-v path.
6. Each pancyclic graph is panconnected.
7. (4.23) Each panconnected. graph is pancyclic.
8. (4.24) If G is a graph of order $n \geq 4$ such that for every pair of nonadjacent vertices u and v we have $d(u) \geq \frac{n}{2}$ and $d(v) \geq \frac{n}{2}$, then G is panconnected.

Problem 4 (20 points): Prove only one problem from each part

Part one:

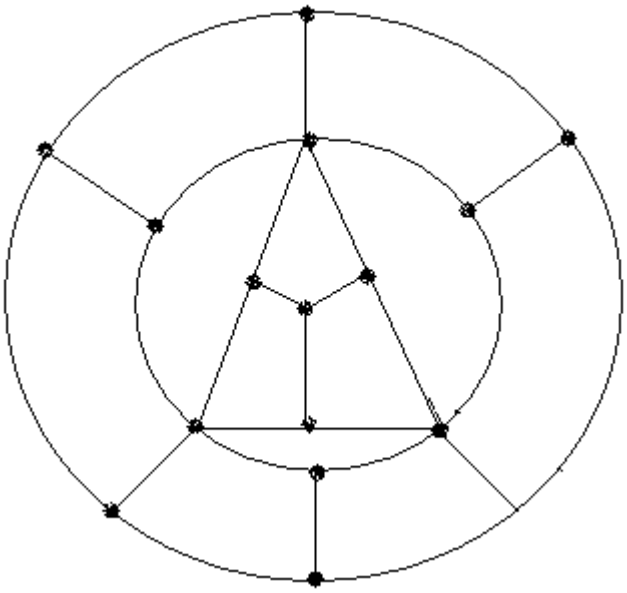
1. (4.22) If a graph *G* is Hamiltonian connected of order $n \geq 4$ then *G* is 3-connected.
2. (3.18) If a graph *G* is *k*- connected then the graph $H = G + K_1$ is $(k+1)$-connected.

Part two:

1. (4.21) If *G* has order *n* and size *m* such that $m \geq \binom{n-1}{2} + 3$, then *G* is Hamiltonian connected.
2. Let *G* be a graph of order *n* , size *m*, and connectivity $\kappa(G) = k$. Prove that $m \geq \frac{kn}{2}$.

Problem 5 (20 points):

1. Determine whether the graph *G* in the figure is Hamiltonian or not



2. Let *N* be a network with source *u* and sink *v* illustrated, where the label on each arc is its capacity.

   (a) Find a minimum cut and find its capacity.
   (b) Assign a flow to *N* so that *val f* = 5..