King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 425- Graph Theory
Final Exam

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SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERS ONLY.

Problem 1 (44 Points) Consider the graph $G = K_{1,2,3}$. Sketch the graph and answer each of the following and justify your answers.

(a) Is G Eulerian graph?

(b) Is G Hamiltonian graph?

(c) Is G nonseparable?

(d) What is the number of edges in a maximum matching of G?

(e) Does G have a perfect matching?

(f) Is G planar? Why?

(g) Give the value of each of the following. (Justification is not required.)

i. $g(G)$, the girth of G

ii. Cen(G), the center of G

iii. $\kappa(G)$, the vertex connectivity of G

iv. $\chi(G)$ the chromatic number of G

v. $\sum_{i=1}^{6} m_{ii}^{(3)}$ where $M = [m_{ij}]$ is the adjacency matrix and $M^3 = [m_{ij}^{(3)}]$

Problem 2 (90 Points): Either prove or disprove each of the following. G is a graph of order n and size m.

(1) Any bipartite graph with an odd number of vertices is not Hamiltonian.

(2) Every transitive tournament is acyclic.

(3) Every maximum matching is perfect matching.

(4) Every Pancyclic graph is Panconnected.

(5) If G has only two vertices of odd order, then it has a path joining them.

(6) If G is an acyclic graph with $m = n - 1$ then G is a tree.

(7) If is planar graph such that each region is quadrilateral (4-gon), then $3m = 2n + 4$.

(8) If $\delta(G) \geq \dfrac{n-1}{2}$ then G is connected.

(9) The Peterson graph is planar.

(10) Any connected graph G has a cycle of length at least $\delta(G) + 1$.

<u>Problem 3 (66 Points)</u>: Do only 6 problems including problem 5.

(1) Draw the tree whose Prufer sequence is (1, 6, 8, 8, 5, 5, 3).

(2) Show that the score sequence s_i of an ***n***-vertex tournament satisfies:

$$\sum_{i=1}^{n} s_i^2 = \sum_{i=1}^{n} (n - 1 - s_i)^2$$

(3) (a) Give an equivalent condition for a non trivial graph to have 1-factor.

(b) Give an equivalent condition for a matching M to be maximal matching. Define all terminology you use.

(c) Give an equivalent condition for a graph to be planar.

(d) State Menger's theorem.

(e) Define *hamiltonian*-connected graph.

(4) Let G be a planar graph with n vertices such that its complement $\overline{G}$ is isomorphic to its dual G^*. Find all possible values of n. (Answer 1, 8)

(5) Let G be a planar graph containing no triangle.

(b) Use Euler formula to show that G has a vertex of degree at most 3.

(c) Use Induction to show that G is 4-colorable.

(6) Show that the Peterson graph has a cycle double cover.

(7) If G has order $n \geq 3$ such that $d(u) + d(v) = n - 1$, then $\text{Diam(G)} \leq 2$.