

Problem 1: (5 points) Show that the function $f(x) = x^5 - 5x^2 + x$ has a real zero in the interval $(1, 2)$

Problem 2: (5 points) Find all values of A and B that make the following function continuous.

$$f(x) = \begin{cases} Ax^2 - 5 & \text{if } x < -1 \\ Bx & \text{if } -1 \leq x < 2 \\ A - Bx & \text{if } 2 \leq x \end{cases}$$

Problem 3: (10 points) If it exists, find the limit. Use the symbols ∞ or $-\infty$ as appropriate.

a) $\lim_{x \rightarrow \infty} \frac{2x\sqrt{x^2+1}}{1-x^2}$

b) $\lim_{x \rightarrow -\infty} \tan^{-1}(x^2 + x^5)$

Problem 4: (5 points) Find the equation of the tangent line to the curve $y = \sqrt{3x+1}$ at $(1, 2)$.

Problem 1: (5 points) Consider the function $f(x) = x^5 - 3x^2 + x + 3$. Show that there is a positive number c such that $f(c) = 17$.

Problem 2: (5 points) Find all values of A and B that make the following function continuous.

$$f(x) = \begin{cases} Ax^2 - 7 & \text{if } x < -1 \\ Bx & \text{if } -1 \leq x < 3 \\ A - Bx & \text{if } 3 \leq x \end{cases}$$

Problem 3: (10 points) If it exists, find the limit. Use the symbols ∞ or $-\infty$ as appropriate.

c) $\lim_{x \rightarrow -\infty} \frac{2x\sqrt{x^2+1}}{1-x^2}$

d) $\lim_{x \rightarrow \infty} \tan^{-1}(x^2 - x^4)$

Problem 4: (5 points) Find the equation of the tangent line to the curve $y = \sqrt{5x-1}$ at $(1, 2)$.