May 5, 2008

Section Number:______
ID Number: _________
Name: ____________________________

Instructions:

• Write neatly and eligibly. You may lose points for messy work.
• Show all your work. No credits for answers without justification.
• All types of calculators and mobiles are not allowed.
• Make sure that you have 13 different problems. (7 pages + cover)

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1. (4 points) Write out the form of partial fraction decomposition of \( \frac{x + 2}{(x - 2)^2(x^2 + 4)^2} \).
   Do not determine the numerical values of the coefficients.

2. (10 points) Use cylindrical shells to set up the integral for finding the volume of the solid obtained by rotating the region in the first quadrant bounded by \( y = x \) and \( y = x^3 \):
   (DO NOT EVALUATE THE INTEGRALS)
   (a) about the \( x \)-axis.

   (b) about the line \( x = 1 \).
3. (6 points) Find the average value of the function $f(x) = x \tan^{-1} x$ on the interval $[-1, 1]$.

4. (6 points) Evaluate $\int \frac{\sqrt{x^2 - 4x}}{x - 2} \, dx$. 
5. Evaluate the following integrals

(a) (5 points) \( \int \sin^2 3x \, dx \).

(b) (5 points) \( \int \sin^2 x \cos^3 x \, dx \).

(c) (7 points) \( \int \frac{dx}{1 - \sin x + \cos x} \) [Hint: you may use the substitution \( t = \tan \frac{x}{2} \)]
6. (8 points) Evaluate \( I = \int \frac{x^2 + 2x - 1}{x^3 - x} \, dx \)

7. (7 points) Compute \( \int_{1}^{\infty} \frac{\ln x}{x^2} \, dx \).
8. (6 points) Determine whether the integral is convergent or divergent. If it converges, evaluate it. \[ \int_{0}^{1} \frac{dx}{3x - 2} \]

9. (6 points) Given the following sequence:

\[
\left( 1 - \frac{1}{2} \right), \left( \frac{1}{2} - \frac{1}{3} \right), \left( \frac{1}{3} - \frac{1}{4} \right), \left( \frac{1}{4} - \frac{1}{5} \right), \ldots
\]

(a) Find the general term of the sequence.

(b) Show that the sequence is convergent.
10. Determine whether the series is convergent or divergent. If it is convergent, find its sum:

(a) (4 points) \[ \sum_{n=1}^{\infty} n \sin \frac{1}{n} \]

(b) (6 points) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1} + 2^n}{3^n} \]

(c) (6 points) \[ \sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)} \]
11. (6 points) Determine whether the series \( \sum_{n=1}^{\infty} \frac{5 - \sqrt{n}}{n^3} \) is convergent or divergent.

12. (8 points) Consider the series \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \)

   (a) Use the integral test to show that the series converges.

   (b) How many terms of the series would you need to add so that the error in the sum is within 0.01? (Write the answer in the form \( n > \ldots \))