Math 102 Quiz # 3 (Q) ID # ____ serial # ____

section # ____ Time: 15 Minutes Marks $\frac{\quad}{9}$

Q1. Evaluate : $\int_0^1 \underset{II}{\sqrt{x}} \; \underset{I}{\tan^{-1}\sqrt{x}} \, dx$

$$\left[\frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x}\right]_0^1 - \int_0^1 \frac{2}{3}x^{3/2}\frac{1}{1+x}\left(\frac{1}{2\sqrt{x}}\right)dx$$

$$\text{''} \qquad -\frac{1}{3}\int_0^1 \frac{x}{x+1}\,dx$$

$$\text{''} \qquad -\frac{1}{3}\int\left(1-\frac{1}{x+1}\right)dx \quad \left[\begin{array}{c}\text{by long}\\ \text{division}\end{array}\right]$$

$$\left[\frac{2}{3}x^{3/2}\tan^{-1}\sqrt{x} - \frac{x}{3} + \frac{1}{3}\ln(1+x)\right]_0^1$$

$$\frac{2}{3}\left(\frac{\pi}{3}\right) - \frac{1}{3} + \frac{1}{3}\ln(2) - 0$$

$$\frac{2\pi}{9} - \frac{1}{3} + \frac{1}{3}\ln(2)$$

Q2. Evaluate : $\int \tan^5 x \sec^4 x \, dx$

$$\int \tan^5 x\,(1+\tan^2 x)\sec^2 x\,dx$$

$$\int \tan^5 x \sec^2 x\,dx + \int \tan^7 x \sec^2 x\,dx$$

$$\frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + c$$

Math 102 Quiz # 3 (b) ID # —— Serial # ——

Section # —— Time: 15 Minutes Marks $\frac{\quad}{9}$

Q: Evaluate: $I = \int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\, dx$

put $x = \sqrt{3}\sec\theta \Rightarrow dx = \sqrt{3}\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-3} = \sqrt{3\sec^2\theta - 3} = \sqrt{3}\tan\theta$

$x = \sqrt{3} \Rightarrow \sqrt{3} = \sqrt{3}\sec\theta \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

$x = 2 \Rightarrow 2 = \sqrt{3}\sec\theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/6$

$$I = \int_{0}^{\pi/6} \frac{\sqrt{3}\tan\theta\,(\cancel{\sqrt{3}}\,\cancel{\sec\theta}\tan\theta\, d\theta)}{\cancel{\sqrt{3}}\,\cancel{\sec\theta}}$$

$$= \sqrt{3}\int_{0}^{\pi/6} \tan^2\theta\, d\theta$$

$$= \sqrt{3}\int_{0}^{\pi/6} (\sec^2\theta - 1)\, d\theta$$

$$= \sqrt{3}\left[-\theta + \tan\theta\right]_{0}^{\pi/6}$$

$$= \sqrt{3}\left[\left(-\frac{\pi}{6} + \tan\frac{\pi}{6}\right) - 0\right]$$

$$= \sqrt{3}\left[\frac{-\pi}{6} + \frac{1}{\sqrt{3}}\right]$$

Math 102 Quiz #3 (C) ID # ____ Serial # ____

Section #: ____ Time: 15 Minutes Marks $\frac{\quad}{9}$

Q: Evaluate: $\int_0^{\pi} \sin^2 x \cos^4 x \, dx$

$$\int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$\frac{1}{8}\int_0^{\pi} (1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\,dx$$

$$\frac{1}{8}\int_0^{\pi} (1-\cos^2 2x)(1+\cos 2x)\,dx$$

$$\frac{1}{8}\int_0^{\pi} \sin^2 2x\,(1+\cos 2x)\,dx$$

$$\frac{1}{8}\int \sin^2 2x\,dx + \frac{1}{8}\int \sin^2 2x \cos 2x\,dx$$

$$\frac{1}{8}\int \frac{1-\cos 4x}{2}\,dx + \left[\frac{1}{8}\cdot\frac{1}{2}\cdot\frac{\sin^3 2x}{3}\right]_0^{\pi}$$

$$\left[\frac{x}{16} - \frac{1}{16}\frac{\sin 4x}{4} + \frac{1}{48}\sin^3 2x\right]_0^{\pi}$$

$$\left[\frac{x}{16} - \frac{1}{64}\sin 4x + \frac{1}{48}\sin^3 2x\right]_0^{\pi}$$

$$\frac{\pi}{16}$$

Math 102 Quiz # 3 (d) ID # ——— Serial #: ———

Section #: ——— Time: 15 minutes Marks: $\frac{\quad}{9}$

Q: Evaluate: (i) $I_1 = \int \tan^5 x \sec x \, dx$ (ii) $I_2 = \int_{\pi/4}^{\pi/2} x \csc^2 x \, dx$

(i)

$$I_1 = \int (\tan^2 x)(\tan^2 x) \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1)^2 \tan x \sec x \, dx$$

$$= \int \sec^4 x \sec x \tan x \, dx + \int \sec x \tan x \, dx - 2\int \sec^2 x \sec x \tan x \, dx$$

$$= \frac{\sec^5 x}{5} + \sec x - \frac{2}{3} \sec^3 x + C$$

(ii) $I_2 = \int x \csc^2 x \, dx = -x \cot x - \int -\cot x \, dx$

$$= \Big[-x \cot x + \ln|\sin x| \Big]_{\pi/4}^{\pi/2}$$

$$= -\frac{\pi}{2}(0) + \ln(1) + \frac{\pi}{4} - \ln \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4} + \frac{1}{2} \ln(2)$$