Math 102 Quiz # 4 (a) ID#: ______ Ser. # ______

Section #: ______ Time : 15 minutes Marks: $\frac{\quad}{9}$

Q1. Evaluate : $I = \int \sqrt{3-2x-x^2}\, dx$

$$\sqrt{-(x^2+2x+1)+4} = \sqrt{4-(x+1)^2}$$

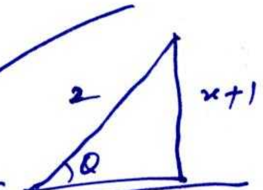
$$I = \int \sqrt{4 - 4\sin^2\theta}\,(2\cos\theta)\,d\theta \qquad \boxed{\begin{array}{l} \text{put } x+1 = 2\sin\theta \\ dx = 2\cos\theta\, d\theta \end{array}}$$

$$= 4\int \cos^2\theta\, d\theta$$

$$= 2\int (1+\cos 2\theta)\, d\theta$$

$$= 2\theta + \sin 2\theta + C$$

$$= 2\sin^{-1}\frac{x+1}{2} + 2\cdot\frac{x+1}{2}\cdot\frac{\sqrt{3-2x-x^2}}{2} + C$$



$$\sqrt{3-2x-x^2} = \sqrt{4-(x+1)^2}$$

$$= 2\sin^{-1}\left(\frac{x+1}{2}\right) + \frac{x+1}{2}\sqrt{3-2x-x^2} + C$$

Q2. Test the sequence $\left\{\frac{\sin 2n}{1+\sqrt{n}}\right\}$ for convergence or divergence. Find its sum, if convergent.

$$|a_n| = \left|\frac{\sin 2n}{1+\sqrt{n}}\right| \le \frac{1}{1+\sqrt{n}}$$

$$\Rightarrow \quad -\frac{1}{1+\sqrt{n}} \le a_n \le \frac{1}{1+\sqrt{n}}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$0\ (\text{as } n\to\infty) \qquad 0\ (\text{as } n\to\infty)$$

$$\Rightarrow \ldots \left\{\frac{\sin 2n}{\ldots}\right\} \to 0 \ :$$

Math 102 Quiz # 4(b) ID #: ______ Ser. # ______

Section #: ______ Time: 15 Minutes Marks: $\overline{\quad 9}$

Q. Test for convergence or divergence. If convergent, then find the sum:

(i) $I = \int_3^\infty \frac{8}{x^2 - 4}\,dx$, $\quad x^2-4=(x-2)(x+2)$ $\qquad$ (ii) $\{n \cos n\pi\}$

(i). $I = \int_3^\infty \left[\frac{2}{x-2} - \frac{2}{x+2}\right]dx$ (By partial fractions)

$= 2\left[\lim_{t\to\infty} \int_3^t \frac{dx}{x-2} - \lim_{t\to\infty} \int \frac{dx}{x+2}\right]$

$= 2\left\{\lim_{t\to\infty} \left[\ln(x-2)\right]_3^t - \left[\ln(x+2)\right]_3^t\right\}$

$= 2 \lim_{t\to\infty} \left[\ln\left(\frac{t-2}{t+2}\right) - 0 + \ln 5\right]$

$= 2 \left[0 + \ln 5\right]$

$= 2 \ln 5$

(ii). $|a_n| = |n \cos n\pi| = |n|\,|(-1)^n| = n$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} n = \infty$

$\Rightarrow \{n \cos n\pi\}$ is divergent.

Math 102 Quiz # 4(C) ID #: ______ Ser. # ______

Section #: ______ Time: 15 Minutes Marks: $\frac{\quad}{9}$

Q1. Evaluate: $\int \frac{e^{2x}}{1+e^x}\,dx$

Put $u = e^x$

$du = e^x\,dx$

$\frac{du}{u} = dx$

$$\int \frac{u^2}{1+u}\,\frac{du}{u}$$

$$\int \frac{u}{1+u}\,du$$

$$\int \left(1 - \frac{1}{1+u}\right)$$

$$u - \ln(1+u) + C$$

$$e^x - \ln(1+e^x) + C.$$

Q2. Determine whether the sequence $\left\{\frac{2n-3}{3n+4}\right\}$ is increasing and bounded. Justify your answer.

Let $f(x) = \frac{2x-3}{3x+4}$ $(x \in \mathbb{R})$

$f'(x) = \frac{17}{(3x+4)^2} > 0 \Rightarrow f(x)$ is increasing

$\Rightarrow f(n) = a_n = \left\{\frac{2n-3}{3n+4}\right\}$ is increasing

$$-\frac{1}{7} = a_1 < a_n = \frac{2n-3}{3n+4} < \frac{2n-3}{3n} < \frac{2n}{3n} = \frac{2}{3}$$

$$-\frac{1}{7} < a_n < \frac{2}{3}$$

$\Rightarrow \left\{\frac{2n-3}{3n+4}\right\}$ is bounded.

Math 102 Quiz # 4(d) ID# ______ Ser. # ______

Section #: ______ Time: 15 minutes Marks: ___ /9

Q. Evaluate: (i) $\int_1^\infty \frac{\ln x}{x^3}\,dx$ (ii) $\int_{-1}^{1} x^{18} \sin x\, dx$

(i).

$$\int_1^\infty \frac{\ln x}{x^3}\,dx$$

$$\lim_{t\to\infty} \int_1^t \frac{\ln x}{x^3}\,dx$$

$$\lim_{t\to\infty} \left(\left[-\frac{1}{2x^2} \ln x \right]_1^t + \frac{1}{2}\int_1^t \frac{1}{x^3}\,dx \right)$$

$$\lim_{t\to\infty} \left[-\frac{1}{2}\frac{\ln t}{t^2} + 0 - \left(\frac{1}{4t^2} + \frac{1}{4}\right)\right]$$

$$\left(\frac{\infty}{\infty}\right)$$

$$\frac{\frac{1}{t}}{2t} = \frac{1}{2t^2}\Big]_{t=\infty} = 0$$

$$\frac{1}{4}$$

(ii). $f(x) = x^{18}$ is even & $g(x) = \sin x$ is odd $\Rightarrow$

$(x^{18})(\sin x)$ is an odd function.

$$\Rightarrow \int_{-1}^{1} x^{18} \sin x\, dx = 0.$$