Exercise 1 (15 points)
Use the reduction of order to find the general solution of the differential equation:

\[ x^2 y'' - xy' + 2y = 0 \quad \text{if} \quad y_1(x) = x \sin(Lnx) \]
Exercise 2 (15 points)
The roots of the characteristic equation of an homogeneous equation with constant coefficients are 1, 2 and $1 + 2i$
1- Find the characteristic equation of this differential equation.

2-Slove the differential equation $2y^{(4)} - 10y^{(3)} + 26y'' - 38y' + 20y = 0$
Exercise 3 (20 points: 10+10)
1-Use the undetermined coefficients method to solve the differential equation
   \[y'' - 5y' + 6y = e^t + 16\] (answers should contain only two unknown constants, No answer with more than two unknown constants is accepted)

2-Use 1) to solve the initial value problem \(y(0) = y'(0) = 0\)
Exercise 4 (15 points)
Use the variation of parameters to solve the differential equation \( y'' - y = \sinh(2x) \)
Exercise 5 (15 points)
Solve the differential equation $x^3 y'' - x^2 y' + 10xy = 0$
Exercise 6 (20 points: 6+6+8)
1. Find the ordinary and singular points of the differential equation
   \[(x - 1)(x^2 + 1)y'' + \tan(x - 1)y' - \sin(x - 1)y = 0\]
2. Find a lower bound for the radius of convergence of power series solutions about the ordinary points \(x_0 = -1\) and \(x_0 = 2\)
3. Find a relation of the coefficients \(C_n\) if \(y(x) = \sum_{n=0}^{\infty} c_n x^n\) is a solution about \(x_0 = 0\) of the differential equation \(y'' + xy' - y = 0\)