Exercise 1 (20 points)

Solve the differential equation \( x \frac{dy}{dx} - y - x^2 \sin x = 0 \)

Solve the initial value problem \( y(\frac{\pi}{2}) = \frac{\pi}{2} \)
Exercise 2 (20 points)
Solve the differential equation \( y(x + y + 1)dx + (x + 2y)dy = 0 \)
by finding an appropriate integrating factor
Exercise 3 (20 points)
Use the undetermined coefficients method to solve the differential equation \( y'' - 2y' + 2y = xe^{2x} \).
Exercise 4 (20 points)

Find a general solution of the differential equation \( x^3 y''' - 6y = 0 \).
Exercise 5 (25 points)
1-Determine the singular points of the differential equation $3xy''+(2-x)y'-y=0$ and classify each one as regular or irregular.
2-Find the indicial equation and the indicial roots and give a power solutions of the differential equation.
Exercise 6 (25 points)

Solve the homogeneous system \( X' = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} X \)
Exercise 7 (30 points)
Use the variation of parameters to solve the nonhomogeneous system

\[
X' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} X + \begin{pmatrix} e^{-t} \\ e^{t} \\ e^{2t} \end{pmatrix}
\]
Exercise 8 (20 points)

Solve the homogeneous system

\[ X' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} X \]
Exercise 9 (20 points)

Use the matrix exponential to solve the system \( X' = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} X + \begin{pmatrix} 1 \\ t \end{pmatrix} \)