| Q 1 | Q 2 | Q 3 | Q 4 | Q 5 | Q 6 | Total mark |
|---|---|---|---|---|---|---|
| | | | | | | |

Table 1: Table of Marks

# King Fahd University of Petroleum and Minerals

## Final Exam for Math 301

## Semester 2, Academic year 2007-2008

**Time allowed 1 hour and 45 minutes**

Full Name: ............................................................

ID Number: ...........................................................

**Note** Important Note

- Show all work
- Having the mobile phone on is prohibited
- Use of programmable calculator is not allowed

**Question 1** (4 POINTS)
a- Expand $f(x) = \sin(\pi x)$ for $-1 < x < 1$ in a Fourier Legendre form.
b- Write the first three terms of the expansion.
**Solution**

**Question 2** (3 POINTS)
a- Show that the Fourier cosine integral representation of

$$f(x) = \begin{cases} 1 & 0 \leq x < 1/2 \\ 1/2 & x = 1/2 \\ 0 & x > 1/2 \end{cases}$$

is given by: $f(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin(\alpha/2)\cos(\alpha x)}{\alpha} d\alpha$.
b- By substituting a suitable value of $x$, show that $\int_0^\infty \frac{\sin\alpha}{\alpha} d\alpha = \frac{\pi}{2}$.
**Solution**

**Question 3** (7 POINTS)
Consider the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial t^2} = 0, \quad x > 0, \quad t > 0 \\ u(0,t) = 0, \quad \lim\limits_{x \longrightarrow \infty} u(x,t) = 0, \quad t > 0 \\ u(x,0) = 1, \quad \frac{\partial u}{\partial t}\Big|_{t=0} = 0, \quad x > 0. \end{cases}$$

For $x > 0$, let $U(x,s)$ be the Laplace transform of $u(x,t)$ with respect to the variable $t$.

a- Show that $U$ satisfies:

$$\begin{cases} U_{xx} - 2U_x - s^2 U = -s, \quad x > 0, \\ U(0,s) = 0, \quad \lim\limits_{x \longrightarrow \infty} U(x,s) = 0, \end{cases}$$

where by $U_{xx}$ and $U_x$ we denote $\frac{\partial^2 U}{\partial x^2}$ and $\frac{\partial U}{\partial x}$ respectively.

b- Solve the boundary value problem in part (a) for $U$.

c- Solve the given the boundary value problem for $u$.

**Solution**

**Question 4** (10 POINTS)

Use the method of separation of variables to solve the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, & 0 < x < 3, \quad t > 0, \\ u(0,t) = 0, \quad u(3,t) = 0, & t > 0, \\ u(x,0) = f(x), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, & 0 < x < 3, \end{cases}$$

where

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 1, & 1 \leq x \leq 2 \\ 3 - x, & 2 \leq x \leq 3. \end{cases}$$

**Solution**

**Question 5** (8 POINTS)
a- Find the Fourier series expansion of the function

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ x, & 0 < x < \pi. \end{cases}$$

b- By substituting a suitable value of $x$ in the obtained expansion, evaluate

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$

**Solution**

**Question 6** (8 POINTS)

a- Use an appropriate Fourier transform to solve the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad x > 0, \quad 0 < y < 2 \\ u(0, y) = 0, \quad 0 < y < 2 \\ u(x, 0) = f(x), \quad u(x, 2) = 0, \quad x > 0. \end{cases}$$

b- Use part (a) to show that $u(1, 0) = \frac{2}{\pi} \int_0^\infty \left( \frac{\sin(\alpha/2)}{\sqrt{\alpha}} \right)^4 d\alpha$ for

$$f(x) = \begin{cases} 1 & 0 < x \leq 1 \\ 0 & x > 1. \end{cases}$$

**Solution**