

Q1- Write down the formula of the Fourier-Bessel series expansion of the function $f(x) = 1, 0 < x < 1$ using Bessel function of order zero that satisfy the condition boundary condition: $J'_0(2\alpha) = 0$.

Q2- Find the eigenfunctions and the equation that defines the eigenvalues of the boundary value problem:

$$y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(1) = 0.$$

Q3- Solve the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, & 0 < x < 1, \quad t > 0 \\ u(0, t) = 0, \quad u(1, t) = 0, & t > 0 \\ u(x, 0) = x(1 - x), & 0 < x < 1. \end{cases}$$