

Q1- Use the method of separation of variables to solve the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, & 0 < x < 1, \quad 0 < y < 1 \\ \frac{\partial u}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial u}{\partial x} \Big|_{x=1} = 0, & 0 < y < 1 \\ u(x, 0) = x, \quad u(x, 1) = 0, & 0 < x < 1. \end{cases}$$

Q2- Use Laplace transform technique to solve the boundary value problem:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, & x > 0, \quad t > 0 \\ u(0, t) = 1, \quad \lim_{x \rightarrow \infty} u(x, t) = 0, & t > 0 \\ u(x, 0) = e^{-x}, \quad \frac{\partial u}{\partial t} \Big|_{t=0} = 0, & x > 0. \end{cases}$$

Solution