

Q 1	Q 2	Q 3	Q 4	Total mark

Table 1: Table of Marks

# King Fahd University of Petroleum and Minerals

Second Major Exam for Math 321

Semester 2, Academic year 2007-2008

**Time allowed 1 hour and 15 minutes**

Full Name: .....

ID Number: .....

Section: .....

### Question 1

Consider the following system:

$$\begin{bmatrix} 10^{-5} & 1 \\ 1 & 2 \end{bmatrix} X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

- a- Use four digits arithmetic Gaussian elimination to solve the given system.
- b- Use partial pivoting to solve the above system.
- c- Compare the obtained results from (a) and (b).
- d- The given system is it ill conditioned or not? Why?

### Question 2

- a) Construct the divided difference-table of the function  $f(x) = |x|^{7/2}$  based on the nodes  $x_i = 0.25(i - 2)$  for  $i = 0, 1, 2, 3, 4$ .
- b) Find the Newton polynomials  $P_2(x)$  which interpolate the function  $f$  at the nodes  $x_2, x_3, x_4$ .
- c) Suppose that  $P_2(x)$  is used to approximate the function  $f(x)$  over the interval  $[0, 0.5]$ . State the form of the error term  $E_2(x) = f(x) - P_2(x)$  for  $x \in [0, 0.5]$ .
- d) Find the error bound of  $E_2(x)$  for any  $x \in [0, 0.5]$ .

### Question 3

Assume that  $f \in C^{n+1}[a, b]$  where  $n \geq 1$  and also  $a = x_0 < x_1 < \dots < x_n = b$ .

- a) Construct the general form of the Lagrange polynomial  $P_n(x)$  interpolating  $f(x)$  at the nodes  $x_0, x_1, \dots, x_n$ .

### Question 4

Consider the system:

$$\begin{cases} 5x - y + z & = 10 \\ 2x + 8y - z & = 11 \\ -x + y + 4z & = 3 \end{cases}$$

Start with  $P_0 = (0, 0, 0)$  and find  $P_k$  using Jacobi iteration for  $k = 1, 2, 3, 4$ . Will  $P_k$  converges to the true solution. Why?