1. (a) Find the $F, F_1$ and $F_2$. Give the domain of analyticity of each in $\alpha$- plane.

(b) Let
\[f(\alpha) = \frac{1}{(\alpha - k \cos \theta)(\alpha + k)^{1/2}}\]
where $k = k_1 + ik_2, k_1, k_2 > 0$.
Find $f(\alpha)$ in the upper (lower) half plane.

2. We define the Mellin transform as
\[M \{f(x)\} = \tilde{\tilde{f}}(p) = \int_0^\infty x^{-1} f(x) dx\]
\[M^{-1}\{\tilde{\tilde{f}}(p)\} = f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-p} \tilde{\tilde{f}}(p) dp\]
Show that
\[M\{xf''(x)\} = -p\tilde{\tilde{f}}(p)\]
\[M\{x^2f''(x)\} = p(p+1)\tilde{\tilde{f}}(p)\]
\[M\{\frac{1}{x}f(\frac{1}{x})\} = \tilde{\tilde{f}}(1-p)\]
\[M\{\log xf(x)\} = \frac{d}{dp}\tilde{\tilde{f}}(p)\]

3. We define the Hankel transform of order $n$ as
\[H_n \{f(x)\} = \tilde{\tilde{f}}_n(k) = \int_0^\infty J_n(kr)f(r)dr,\]
where $J_n$ is the Bessel function.
Show that
\[H_0\{\frac{1}{r} \frac{d}{dr} (r \frac{df}{dr})\} = -k^2 \tilde{\tilde{f}}_0(k).\]
Solve
\[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, 0 < r < \infty, t > 0\]
\[u(r,0) = f(r), 0 < r < \infty\]
\[u_t(r,0) = g(r), 0 < r < \infty.\]