King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101- Calculus I
Exam I
2007-2008 (073)

Tuesday, July 22, 2008                                         Allowed Time: 2 hours

Name: _______________________________________________________

ID Number: ____________

Section Number:___________                   Serial Number:___________

Instructions:
1. Write neatly and eligibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**
4. Make sure that you have 8 different problems (6 pages + cover page)

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<th>Problem No</th>
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1. Evaluate the limit if it exists. Justify your answer

(a) \( \lim_{x \to 0^+} \frac{x - 1}{x^2 + 2x} \). (4 pts.)

(b) \( \lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}} \). (8 pts.)

(c) \( \lim_{x \to 0^+} x \sin \left( \frac{\sqrt{x} + 2}{x} \right) \). (6 pts.)
(d) \( \lim_{x \to 1} \arcsin \left( \frac{1 - x}{1 - x^2} \right) \). (4 pts.)

(e) \( \lim_{x \to -\infty} \frac{x^3 - 2x + 7}{-2x^2 + x - 3} \). (4 pts.)

(f) \( \lim_{x \to +\infty} \left( \sqrt{9x^2 + x - 3x} \right) \). (7 pts.)
2. Use the graph of \( f(x) = \sqrt{x-1} \) to find a number \( \delta \) such that \( |\sqrt{x-1} - 1| < 0.1 \) whenever \( |x-2| < \delta \). (7 pts.)

3. Where is the function \( f(x) = \frac{1}{1 - e^{-x}} \) continuous? (7 pts.)
4. Find the constant $k$ that makes the function continuous on $(-\infty, +\infty)$.

\[ f(x) = \begin{cases} 
  x^2 - k^2 & \text{if } x \leq 2 \\
  kx + 5 & \text{if } x > 2
\end{cases} \]

5. Show that the equation $x \ln x = \sin x$ has a root in the interval $(1, e)$. 

(8 pts.)
6. (a) How many horizontal asymptotes can a function have? Illustrate your answer graphically. (6 pts.)

(b) Does the graph of \( f(x) = \ln(9-x^2) \) have a vertical asymptote \( x = 3 \). Justify. (7 pts.)

(ii) at \( x = -1 \). Justify.

7. The position function of a particle moving in a straight line is given by the equation of motion \( s(t) = \frac{1-t}{1+t} \), where \( t \) is measured in seconds and \( s \) in meters. Find the instantaneous velocity of the particle when \( t = 1 \). (8 pts.)
8. (a) **TRUE** or **FALSE**. Justify: If \( f'(a) \) exists, then \( \lim_{x \to a} f(x) = f(a) \). (4 pts.)

(b) Is \( f(x) = x \cdot |x| \) differentiable at \( x = 0 \). Justify. (6 pts.)

(c) Graph the derivative of the function whose graph is given below. (6 pts.)