King Fahd University of Petroleum & Minerals  
Department of Mathematics and Statistics  
2007-2008 (073)  
Calculus III (MATH 201)  
Final Exam

INSTRUCTIONS

1. For Questions 1 to 3 show your work.
2. For Questions 4 to 12 circle the correct answer in the table below.
3. Write clearly and legibly. Marks may be deducted for messy work.

<table>
<thead>
<tr>
<th>Question</th>
<th>Marks out of</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/ 6</td>
</tr>
<tr>
<td>2</td>
<td>/ 6</td>
</tr>
<tr>
<td>3</td>
<td>/ 6</td>
</tr>
<tr>
<td>4</td>
<td>A B C D E</td>
</tr>
<tr>
<td>5</td>
<td>A B C D E</td>
</tr>
<tr>
<td>6</td>
<td>A B C D E</td>
</tr>
<tr>
<td>7</td>
<td>A B C D E</td>
</tr>
<tr>
<td>8</td>
<td>A B C D E</td>
</tr>
<tr>
<td>9</td>
<td>A B C D E</td>
</tr>
<tr>
<td>10</td>
<td>A B C D E</td>
</tr>
<tr>
<td>11</td>
<td>A B C D E</td>
</tr>
<tr>
<td>12</td>
<td>A B C D E</td>
</tr>
<tr>
<td>TOTAL</td>
<td>/ 72</td>
</tr>
</tbody>
</table>
1. Find the length of the polar curve $r = \sqrt{1 + \cos 2\theta}, \; 0 \leq \theta \leq \pi$.

2. Let $z = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$. Evaluate
$$\left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 - \left( \frac{\partial z}{\partial r} \right)^2 - \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2.$$
3. The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most 1.3 cm in each. Use differentials to estimate the maximum error in the calculated length of the hypotenuse.

4. The distance between the skew lines \(L_1: x = 1 + t, y = -1 + 3t, z = 1\) and \(L_2: x = 1 + t, y = 1 + t, z = 3\) is

(a) 1   (b) 2   (c) 3   (d) 4   (e) 5
5. The solid region $S$ is in the first octant and is bounded by the coordinates plane and the plane $x/a + y/b + z/c = 1$, where $a$, $b$, $c$ are positive constants. If $S$ has density $\rho(x, y, z) = 4x$, then the mass of $S$ is

(a) $\frac{a^2bc}{2}$
(b) $\frac{a^2bc}{3}$
(c) $\frac{a^2bc}{4}$
(d) $\frac{a^2bc}{6}$
(e) $\frac{a^2bc}{8}$

6. By reversing the order of integration, we find that

$$\int_0^2 \frac{y}{\sqrt{x^7 + 16}} \, dx \, dy =$$

(a) $\frac{20}{7}$
(b) $\frac{16}{7}$
(c) $\frac{12}{7}$
(d) $\frac{8}{7}$
(e) $\frac{4}{7}$

7. The volume of the solid region between the sphere $\rho = \cos \phi$ and the surface $z = \sqrt{4 - x^2 - y^2}$ is

(a) $\frac{31\pi}{6}$
(b) $\frac{25\pi}{6}$
(c) $\frac{19\pi}{6}$
(d) $\frac{13\pi}{6}$
(e) $\frac{7\pi}{6}$
8. The function \( f(x, y) = 9x^3 + y^3/3 - 4xy \) has exactly
(a) one saddle point, one local minimum, and no local maximum
(b) one saddle point, one local maximum, and no local minimum
(c) one saddle point, one local minimum, and one local maximum
(d) one saddle point, no local maximum, and no local minimum
(e) one local minimum, one local maximum, and no saddle point.

9. Lagrange multipliers are used to find the maximum \( A \) and the minimum \( B \) of \( f(x, y, z) = xy + z^2 \)
on the curve of intersection of the plane \( x - y = 0 \) and the sphere \( x^2 + y^2 + z^2 = 4 \). Then \( A + B = \)
(a) 8  (b) 6  (c) 4  (d) 2  (e) 0

10. By converting to polar coordinates, we find that \[ \int_{0}^{2} \int_{0}^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dy dx = \]
(a) \( \frac{\pi}{2} + 2 \)  (b) \( \frac{\pi}{2} + 1 \)  (c) \( \frac{\pi}{4} + 2 \)  (d) \( \frac{\pi}{4} + 1 \)  (e) \( \frac{\pi + 1}{2} \)
11. The line of intersection of the planes $2x + y + 4z = 8$ and $x + 3y - z = -1$ meets the plane $x + y + z = 3$ at the point $A$. The distance from $A$ to the origin is

(a) $\sqrt{29}$  (b) $\sqrt{26}$  (c) $\sqrt{23}$  (d) $\sqrt{20}$  (e) $\sqrt{17}$

12. The hyperboloid $x^2 - y^2 + 2z^2 = 1$ contains the point $P(a, b, c)$. If the normal line to the hyperboloid at the point $P$ is parallel to the line that joins the points $(3, -1, 0)$ and $(5, 3, 6)$, then $a^2 + b^2 + c^2 =$

(a) $41/6$  (b) $35/6$  (c) $29/6$  (d) $23/6$  (e) $17/6$