Math 101

Quiz 7

Ex1. Find the derivative of the function:

a) $y = \tan^{-1}(\sin^{-1} x)$

b) $y = \sec^{-1}(\sqrt{x})$

Ex2. Find y' by implicit differentiation:

a) $y^4 + 4y - 3x^3 \sin(y) = 2x + 1$

b) $x^2 y = \cos^{-1}(xy)$

Math 101 Quiz 7 Solutions

Ex1. a) $y = \tan^{-1}(\sin^{-1}x) = f(g(x))$, with $f(x) = \tan^{-1}x$ and $g(x) = \sin^{-1}x$. Using the chain rule, we have:

$$\frac{dy}{dx} = g'(x)\, f'(g(x)) = \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{1+(\sin^{-1}x)^2} = \frac{1}{\sqrt{1-x^2}\,\left[1+(\sin^{-1}x)^2\right]}$$

b) $y = \sec^{-1}(\sqrt{x}) = f(g(x))$, with $f(x) = \sec^{-1}x$ and $g(x) = \sqrt{x}$

By the Chain rule, we obtain that:

$$\frac{dy}{dx} = g'(x)\, f'(g(x)) = \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{x}\sqrt{x-1}} = \frac{1}{2x\sqrt{x-1}}$$

Ex2. a) $y^4 + 4y - 3x^3\sin y = 2x+1 \Leftrightarrow \frac{d}{dx}(y^4+4y-3x^3\sin y) = \frac{d}{dx}(2x+1)$

$$\Leftrightarrow 4y'y^3 + 4y' - 9x^2\sin y - 3x^3y'\cos y = 2$$

$$\Leftrightarrow y'(4y^3+4-3x^3\cos y) = 2 + 9x^2\sin y$$

$$\Leftrightarrow \quad y' = \frac{2+9x^2\sin y}{4y^3+4-3x^3\cos y}$$

b) $x^2y = \cos^{-1}(xy) \Leftrightarrow \frac{d}{dx}(x^2y) = \frac{d}{dx}\left(\cos^{-1}(xy)\right)$

$$\Leftrightarrow 2xy + x^2y' = -(y+xy')\frac{1}{\sqrt{1-x^2y^2}}$$

$$\Leftrightarrow x^2y' + \frac{xy'}{\sqrt{1-x^2y^2}} = -2xy - \frac{y}{\sqrt{1-x^2y^2}}$$

$$\Leftrightarrow y'x\left(x + \frac{1}{\sqrt{1-x^2y^2}}\right) = -y\left(2x + \frac{1}{\sqrt{1-x^2y^2}}\right)$$

$$\Leftrightarrow \quad y' = -\frac{y}{x}\,\frac{2x + \frac{1}{\sqrt{1-x^2y^2}}}{x + \frac{1}{\sqrt{1-x^2y^2}}}$$