Math 101

Quiz 3

Ex 1. Let $f(x) = \begin{cases} e^x & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$

a) is f continuous at 0? Justify your answer.

b) Find the domain of continuity of f.

Ex 2. Prove that the polynomial function $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root in the interval $[-1, 2]$.

Math 101

Quiz 3 Solutions

Ex1. a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x = 1$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$

Thus $\lim_{x\to 0} f(x)$ does not exists since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$
Therefore f cannot be continuous at 0.

b) f is continuous on $(-\infty, 0)$, because on the later f is an exponential function. Also on $(0, +\infty)$ f is continuous because f is a polynomial function on $(0, +\infty)$. Hence f is continuous on $(-\infty, 0) \cup (0, +\infty)$.

Ex2. We want to prove the existence of an x_0 in $[-1, 2]$ such that $p(x_0) = 0$. To do this, we will use the IVT theorem.

First we verify the hypothesis of the IVT theorem:

*) The polynomial function p is continuous on $[-1, 2]$

*) $p(-1) = 8$ and $p(2) = -21$

*) $p(2) < 0 < p(-1)$

Hence the IVT theorem (or Bolzano's theorem) implies the existence of an x_0 in $(-1, 2)$ such that $p(x_0) = 0$.