1. (10 points) Find each of the following limits of the function $f$ whose graph is given in the adjacent figure

(a) $\lim_{x\to -2^-} f(x) =$

(b) $\lim_{x\to -1} f(x) =$

(c) $\lim_{x\to 0^-} f(x) =$

(d) $\lim_{x\to 0^+} f(x) =$

(e) $\lim_{x\to 1} f(x) =$

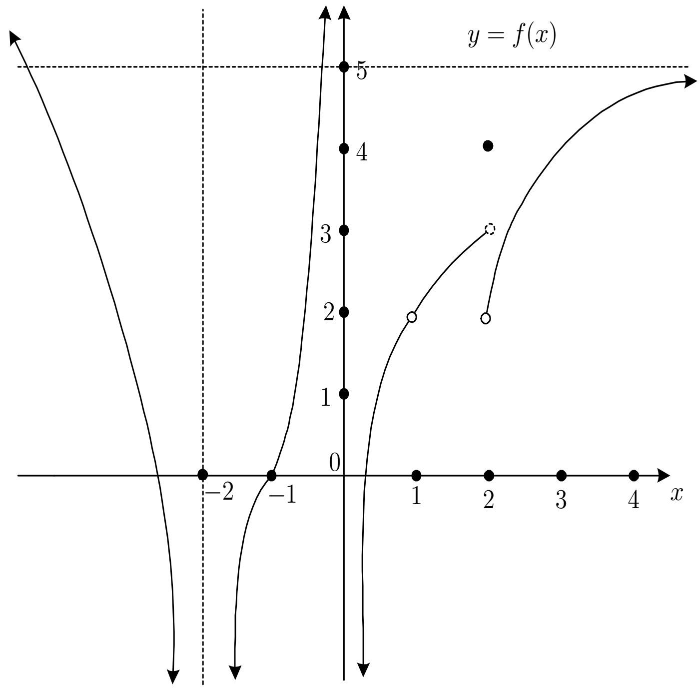
(f) $\lim_{x\to 2^-} f(x) =$

(g) $\lim_{x\to 2^+} f(x) =$

(h) $\lim_{x\to 2} f(x) =$

(i) $\lim_{x\to -\infty} f(x) =$

(j) $\lim_{x\to +\infty} f(x) =$




2. (7 points) Sketch the graph of an example of a function $f$ that satisfies the following conditions:

(a) $f'(-3) = f'(3) = 0$,

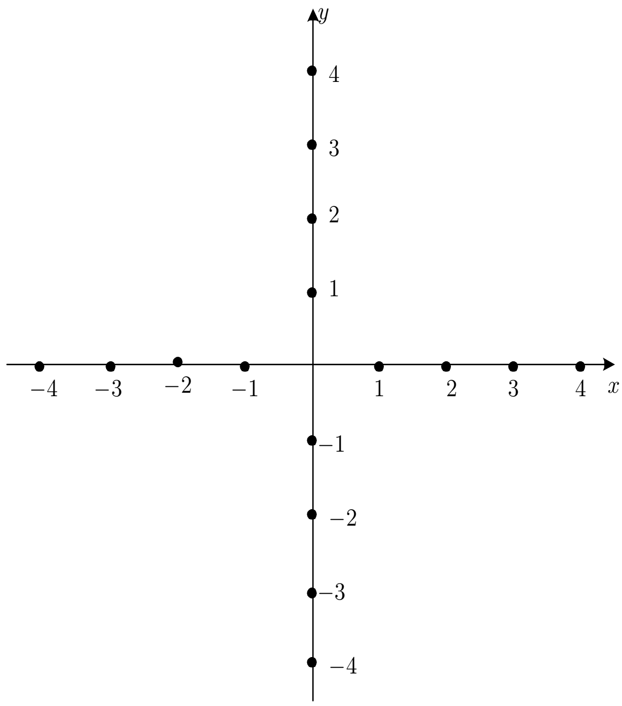
(b) $\lim_{x\to 0^-} f(x) = -1$,

(c) $\lim_{x\to 0^+} f(x) = 1$,

(d) $f(0)$ is undefined,

(e) $\lim_{x\to 2} f(x) = -1$,

(f) $f(2) = 1$.



3. Evaluate each of the following limits (show your steps).

(a) (3 points) $\lim_{x\to 2} \frac{x^2 - 3x + 2}{2 - x}$.

(b) (4 points) $\lim_{x\to +\infty} \frac{1 - x - 2x^3}{x^3 + 2x^2 + 1}$.

(c) (4 points) $\lim_{x\to -\infty} \frac{\sqrt{3x^2 + 7}}{4x - 11}$.

(d) (4 points) $\lim_{x\to \frac{1}{2}^-} \frac{12x^2 - 6x}{|2x - 1|}$.

4. (4 points) If $\lim_{x\to 2} f(x) = 7$ and $\lim_{x\to 2} g(x) = 3$, find $\lim_{x\to 2} \dfrac{\sqrt{x+f(x)}}{|x-2|-(g(x))^2}$. **Justify each step.**

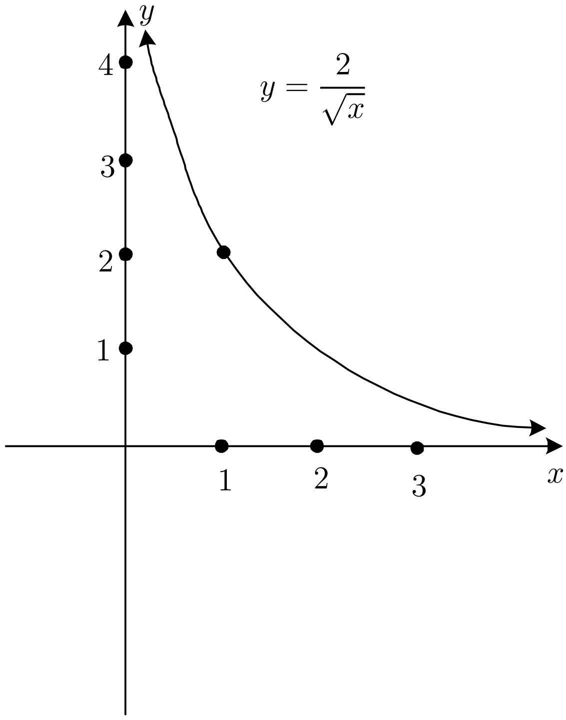
5. (10 points) Use the Squeeze Theorem to show that $\lim_{x\to 0} \sin x \cdot \cos\dfrac{1}{x} = 0$.

6. The displacement (in meters) of a particle moving in a straight line is given by the equation $S = 40 + 16t^2$, where $t$ is measured in seconds.

(a) (3 points) Find the average velocity of the particle over the time interval with endpoints between $1$ and $1+h$.

(b) (2 points) Use part (a) to find the instantaneous velocity of the particle when $t = 1$.

7. (9 points) Use the graph of $f(x) = \dfrac{2}{\sqrt{x}}$ to find the largest a number $\delta$ such that $|f(x) - 2| < \dfrac{1}{2}$ whenever $0 < |x - 1| < \delta$. $\left(\text{Show your steps and write your answer in a rational form } \dfrac{p}{q}\right)$.



8. (8 points) Find an equation of the tangent line to the curve $f(x) = \dfrac{2}{x+3}$ at the point where $x = -1$. [You must use limits].

9. (9 points) If $[x]$ denotes the greatest integer less than or equal to $x$, find all values of $x$ for which the following function is continuous:

$$f(x) = \begin{cases} [x], & \text{if } -2 \le x < 0 \\ x, & \text{if } 0 \le x < 1 \\ 3x - 2, & \text{if } 1 \le x \le 2 \end{cases}$$

(Use limits to justify your answers).

10. (6 points) Determine whether the function

$$f(x) = \frac{\sqrt{2x+9} - \sqrt{x+9}}{2x}$$

has a removable discontinuity, a jump discontinuity, or an infinite discontinuity at $x = 0$.

11. (5 points) Use the Intermediate Value Theorm to show that there is a root of the equation $x^6 + x^4 - 1 = 0$ in the interval $[-1, 1]$.

12. (4 points) The limit $\lim_{x \to \frac{\pi}{2}} \frac{6(\sin x - 1)}{2x - \pi}$ represents the derivative of some function $f$ at some number $a$. State such an $f$ and $a$. (give a reason to your answer)

13. (8 points) Find the equations of all horizontal asymptotes to the graph of $f(x) = \tan^{-1}(e^{-2x} - 1)$. (Show your work)