MATH 102 - Quiz 5 Section number: Student ID:

Instructions: You are required to attempt all questions. Each is worth 10 points.

1. Find the radius of convergence and the interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{(x+2)^n}{n2^n}$.

Solution:
Let $a_n = (-1)^n \frac{(x+2)^n}{n2^n}$. Convergence occurs when $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L < 1$.

$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$

$\Rightarrow \lim_{n\to\infty} |\frac{(x+2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x+2)^n}|$

$\Rightarrow \lim_{n\to\infty} |\frac{(x+2)}{2} \cdot \frac{n}{n+1}| = |\frac{x+2}{2}| < 1.$

So the interval of convergence is $-4 < x < 0$. We need to check at the end points, i.e. at $x = 0$ and $x = -4$.
At $x = 0$, the series converges by the alternating series test. At $x = -4$, the resulting series diverges by the p-test. Hence, the interval of convergence is $(-4, 0]$.

2. Find the Taylor series for $f(x) = ln(x)$ centered at $a = 2$. Assume that $f(x)$ has a power series expansion.

Solution:
After working through for the first three derivatives, you can derive this relation: $f^{(n)}(x) = (-1)^{n-1}(n-1)!x^{-n}$.

$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(2)(x-2)^n$

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{n2^n}(x-2)^n$

3. Find the arc length function for the curve $y = 2x^{3/2}$ with starting point $P_o(1, 2)$

Solution:
$f(x) = 2x^{3/2} \Rightarrow f'(x) = 3x^{1/2}$.

$L(x) = \int_a^x (\sqrt{1 + f'(t)^2}dt = \int_1^x \sqrt{1+9t}dt$

Now we can use the substitution rule. Set $u = 1 + 9t$.

Then, we will have $\int \sqrt{u}\frac{du}{9} = \frac{2u^{3/2}}{27} \rightarrow \frac{2(1+9t)^{3/2}}{27}$

$L(x) = [\frac{2(1+9t)^{3/2}}{27}]_1^x = \frac{2}{27}((1+9x)^{3/2} - 10^{3/2})$.