QUIZ♯2 Math102-sec01.

Net Time Allowed: 20 minutes

Name: **ID ♯ :** **section:**

Exercise1:

Let f be a continuous function on $[1,x]$ with $f(1)=0$, F, G two differentiable functions such that:
$F(x)=\int_1^x f(t)\,dt$ and $G(u)=\int_1^u e^{-t}F(t)\,dt$.
Find $G'(1)+G''(1)$.

solution:

$$G'(u) = e^{-u}F(u) \Longrightarrow G'(1) = e^{-1}\overset{0}{F(1)} = 0$$

$$G''(u) = -e^{-u}\overset{0}{F(u)} + e^{-u}F'(u) = e^{-u}\big(f(u)-F(u)\big) \Rightarrow G''(1)=0$$

$$\left(G''(1) = e^{-1}\big(\overset{0}{f(1)} - \overset{0}{F(1)}\big) = 0\right) \Longrightarrow \boxed{G'(1)+G''(1) = 0+0 = 0}$$

Exercise2:

Evaluate $I=\int \frac{x-1}{\sqrt{1-x^2}}\,dx$.

solution:

$$I = \int \frac{x-1}{\sqrt{1-x^2}}\,dx = \int \frac{x\,dx}{\sqrt{1-x^2}} - \int \frac{dx}{\sqrt{1-x^2}}$$

or $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C_1$

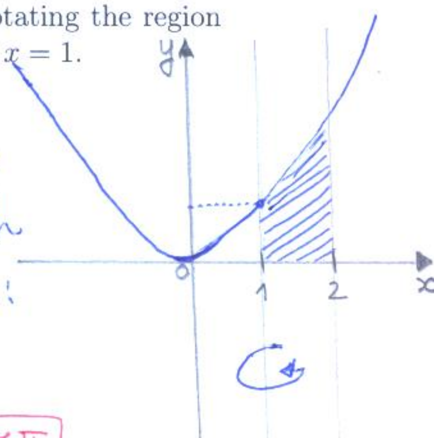
$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx \overset{u=1-x^2}{=} -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\times 2\sqrt{u} + C_2$$

$$\Longrightarrow \boxed{I = -\sqrt{1-x^2} + \sin^{-1}x + C}$$

Exercise3:

Use the method of shell to find the volume of the solid obtained by rotating the region bounded by $y=x^2$, $y=0$, $x=1$, $x=2$ rotated about the line $x=1$.

solution:



Set $u = x-1$, so Rotating about The line $x=1$ is equivalent to rotating about $u=0$ (which is The Same as y-axis), so By shell method:

$$V = \int_{\substack{x=1\\u=0}}^{\substack{u=1\\x=2}} 2\pi u(u+1)^2\,du = 2\pi\int_0^1 (u^3+2u^2+u)\,du$$

$$V = 2\pi\left[\frac{1}{4}u^4 + \frac{2}{3}u^3 + \frac{1}{2}u^2\right]_0^1 = 2\pi\left(\frac{1}{4}+\frac{2}{3}+\frac{1}{2}\right) \Longrightarrow \boxed{V = \frac{17\pi}{6}}$$