**MATH 102 - Quiz 1** Section number: Student ID:

Instructions: You are required to attempt all questions. Each is worth 10 points.

1. Determine a region whose area is equal to the given limit
$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}(5+\frac{2i}{n})^{10}$

Either Solution 1:
Width of interval is $\Delta x = \frac{2}{n}$. So the gap is 2. $x_i = 5 + (\Delta x)i \Rightarrow x \in [5, 7]$.
$f(x) = x^1 0$

or Solution 2:
Width of interval is $\Delta x = \frac{2}{n}$. So the gap is 2. $x_i = (\Delta x)i \Rightarrow x \in [0, 2]$.
$f(x) = (x+5)^1 0$

2. Find the derivative of the function $g(x) = \int_{tan(x)}^{x^2} \frac{1}{\sqrt{2+t^4}}$

Solution:
To do this problem, we need to apply the chain rule in addition to the Fundamental Theorem of Calculus. Suppose $f(x) = \frac{1}{\sqrt{2+x^4}}$.

If $F'(x) = f(x)$ and let $\alpha(x) = tan(x)$ and $\beta(x) = x^2$, then $g(x) = F(\beta(x)) - F(\alpha(x))$.

$g'(x) = F'(\beta(x)) \times \beta'(x) - F'(\alpha(x)) \times \alpha'(x)$

$g'(x) = f(\beta(x)) \times \beta'(x) - f(\alpha(x)) \times \alpha'(x)$

$g'(x) = \frac{2x}{\sqrt{2+x^8}} - \frac{sec^2(x)}{\sqrt{2+tan^4(x)}}$

3. If $f(x)$ is continuous and $\int_0^9 f(x)dx = 4$, then find $\int_0^3 xf(x^2)dx$

Solution:
let $u = x^2$, then $\frac{du}{2} = xdx$.

$$\int_0^3 xf(x^2)dx = \frac{1}{2}\int_{0^2}^{3^2} f(u)du = \frac{4}{2} = 2.$$