1. The curve $C: x = t - \ln t,\ y = t + \ln t$ is concave down on

(a) $(1, \infty)$

(b) $(0, 1)$

(c) $(0, \infty)$

(d) $(-\infty, 0) \cup (1, \infty)$

(e) $(-\infty, 1)$

2. The slope of the tangent line to the polar curve $r = 1 + \sin\theta$ at $\theta = \dfrac{\pi}{4}$ is

(a) $-\dfrac{\sqrt{2}+2}{\sqrt{2}}$

(b) $-\dfrac{1}{\sqrt{2}}$

(c) $1 + \dfrac{\sqrt{2}}{2}$

(d) $\dfrac{\sqrt{2}-2}{\sqrt{2}}$

(e) $\dfrac{\sqrt{2}}{2-\sqrt{2}}$

3. The area of the region that lies inside both curves $r = 4\cos\theta$ and $r = 4\sin\theta$ is

(a) $2\pi - 4$

(b) $2\pi + 4$

(c) 4π

(d) $\pi + 2$

(e) $\pi - 2$

4. Vector projection of $\vec{u} = \langle 1, 2, 3 \rangle$ onto $\vec{v} = \langle 1, 4, 0 \rangle$ is

(a) $\langle \frac{9}{17}, \frac{36}{17}, 0 \rangle$

(b) $\langle \frac{9}{\sqrt{17}}, \frac{36}{\sqrt{17}}, 0 \rangle$

(c) $\langle \frac{9}{14}, \frac{36}{14}, 0 \rangle$

(d) $\langle \frac{9}{17}, \frac{18}{17}, \frac{27}{17} \rangle$

(e) $\langle \frac{9}{14}, \frac{18}{14}, \frac{27}{14} \rangle$

5. The value of k for which the vectors $\vec{a} = \langle 1, 4, -7 \rangle$, $\vec{b} = \langle 2, -1, 4 \rangle$ and $\vec{c} = \langle k, 0, 1 \rangle$ are coplanar is

(a) $k = 1$

(b) $k = -1$

(c) $k = -\dfrac{9}{23}$

(d) $k = \dfrac{7}{23}$

(e) $k = \dfrac{1}{9}$

6. Suppose that L_1 is the line passing through $(1, 0, 3)$ and $(0, 0, 4)$ and L_2 is the line passing through $(1, 0, 1)$ with direction vector $\vec{v} = \langle 3, -1, 1 \rangle$. Then

(a) L_1 and L_2 are skew lines

(b) L_1 and L_2 are parallel lines

(c) L_1 and L_2 are perpendicular lines

(d) L_1 and L_2 intersect at $(4, -1, 2)$

(e) L_1 and L_2 are identical

7. An equation of the plane $(P1)$ that passes through the line of intersection of the planes $(P2)$ $x - z = 1$ and $(P3)$ $y + 2z = 3$, and is perpendicular to the plane $(P4)$ $x + y - 2z = 1$ is

(a) $x + y + z = 1$

(b) $x + 2y = 9$

(c) $x + y = 4$

(d) $3x - y + z = 1$

(e) $x - 2y + z = -5$

8. The quadric surface $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$ represents

(a) Circular cone with vertex at $(2, -1, 1)$ and axis parallel to y-axis

(b) Ellipsoid with center $(-2, -1, 1)$

(c) Elliptic cone with vertex at $(1, -1, 1)$ and axis parallel to z-axis

(d) Circular cone with vertex at $(2, 1, 1)$ and axis parallel to y-axis

(e) Circular paraboloid with vertex at $(-4, -2, -2)$ and axis parallel to z-axis

9. Let $f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & (x,y) \neq (0,0) \\ 3 & (x,y) = (0,0) \end{cases}$

and $L = \lim_{(x,y)\to(0,0)} f(x,y)$. Then

(a) L does not exist

(b) $L = 3$

(c) $L = 0$ and $f(x,y)$ is not continuous at $(0,0)$.

(d) $L = 1$ and $f(x,y)$ is not continuous at $(0,0)$.

(e) $L = 3$ and $f(x,y)$ is not continuous at $(0,0)$.

10. If $u = e^{ax+by+cz}$, where $a^2 + b^2 + c^2 = 6$, then $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}$ is equal to

(a) $6u$

(b) u

(c) $\dfrac{6}{u}$

(d) $6u^2$

(e) u^2

11. Let $z = f(x, y), x = r\cos\theta, y = r\sin\theta$. Then $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$ is equal to

(a) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$

(b) $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$

(c) $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{1}{x^2 + y^2}\left(\frac{\partial z}{\partial y}\right)^2$

(d) $\left(\frac{\partial z}{\partial x}\right)^2 \cdot \left(\frac{\partial z}{\partial y}\right)^2$

(e) $(x^2 + y^2)\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$

12. Using the linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$, the value of $\sqrt{(2.9)^2 + (4.1)^2}$ is approximately equal to

(a) $\frac{251}{50}$

(b) $\frac{257}{50}$

(c) $\frac{1}{50}$

(d) $\frac{6}{5}$

(e) $\frac{3}{25}$

13. The directional derivative of $f(x,y) = x^2 + \sin(xy)$ at the point $(1,0)$ is equal to 1 in the direction of the unit vectors

(a) $\langle 0,1\rangle$ and $\langle \frac{4}{5}, -\frac{3}{5}\rangle$

(b) $\langle 1,0\rangle$ and $\langle \frac{4}{5}, -\frac{3}{5}\rangle$

(c) $\langle -1,0\rangle$ and $\langle -\frac{4}{5}, \frac{3}{5}\rangle$

(d) $\langle 1,0\rangle$ and $\langle -\frac{4}{5}, -\frac{3}{5}\rangle$

(e) $\langle 0,1\rangle$ and $\langle -\frac{4}{5}, -\frac{3}{5}\rangle$

14. The function $f(x,y) = x^4 + y^4 - 4xy + \sqrt{5}$ has

(a) Local minimum at $(1,1), (-1,-1)$ and saddle point at $(0,0)$

(b) Local minimum at $(1,1), (-1,-1), (1,-1), (-1,1)$ and saddle point at $(0,0)$

(c) Local maximum at $(1,1), (-1,-1)$ and saddle point at $(0,0)$

(d) Local minimum at $(-1,-1)$, local maximum at $(1,1)$ and saddle point at $(0,0)$

(e) Local minimum at $(1,1)$, local maximum at $(-1,-1)$ and saddle point at $(0,0)$

15. Determine the nature of the critical points $(1,2), (-2,3)$, and $(-1,-1)$ of the function $g(x,y)$ if

$$\begin{array}{ccc} g_{xx}(1,2)=2 & g_{yy}(1,2)=3 & g_{xy}(1,2)=2 \\ g_{xx}(-2,3)=-4 & g_{yy}(-2,3)=5 & g_{xy}(-2,3)=4 \\ g_{xx}(-1,-1)=-3 & g_{yy}(-1,-1)=-4 & g_{xy}(-1,-1)=3 \end{array}$$

(a) Local minimum at $(-1,-1)$, Local minimum at $(1,2)$, Saddle point at $(-2,3)$.

(b) Local maximum at $(1,2)$, Local minimum at $(-1,-1)$, Saddle point at $(-2,3)$.

(c) Local maximum at $(-2,3), (-1,-1)$, Local minimum at $(1,2)$.

(d) Local minimum at $(1,2)$, Saddle point at $(-2,3)$.

(e) Local minimum at $(-1,-1)$, Local maximum at $(1,2)$, $(-2,3)$.

16. The maximum value of $f(x,y,z) = x + 2y - 3z$ subject to the constraint $z = 4x^2 + y^2$ is equal to (Hint: Use Lagrange Multipliers)

(a) $\frac{17}{48}$

(b) 0

(c) $\frac{7}{8}$

(d) 5

(e) -2

17. The volume of the solid that lies under the paraboloid $z = 2b^2 x^2 + a^2y^2$ $(a, b > 0)$ and above the rectangle $[0, a] \times [0, b]$ is

(a) $(ab)^3$

(b) $(a + b)^3$

(c) $a^2b + ab^2$

(d) $a^3 + b^3$

(e) 1

18. The volume of the solid bounded by the surface $z = x\sqrt{x^2 + y}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, and $z = 0$ is

(a) $\frac{2}{15}\left(2^{\frac{5}{2}} - 2\right)$

(b) $\frac{2}{15}\left(2^{\frac{5}{2}} + 2\right)$

(c) $\frac{2}{15}\left(2^{\frac{5}{2}} - 1\right)$

(d) $\frac{3}{15}\left(2^{\frac{5}{2}} + 2\right)$

(e) $\frac{4}{15}\left(2^{\frac{5}{2}} - 1\right)$

19. The volume of the solid under the surface $z = 2x + y^2$ and above the region in xy-plane bounded by $x = y^2$ and $x = y^3$ is

(a) $\dfrac{19}{210}$

(b) $\dfrac{18}{210}$

(c) $\dfrac{1}{7}$

(d) $\dfrac{2}{5}$

(e) $\dfrac{13}{42}$

20. The value of the iterated integral $\displaystyle\int_0^2 \int_{2x}^4 e^{y^2}\, dy\, dx$ is equal to

(a) $\dfrac{1}{4}\left(e^{16} - 1\right)$

(b) $\dfrac{1}{2}\left(e^{16} + 1\right)$

(c) $\dfrac{1}{4}\left(e^{16} + 1\right)$

(d) $\dfrac{1}{2}\left(e^{16} - 1\right)$

(e) $\dfrac{1}{8}\left(e^{16} - 2\right)$

21. The value of the iterated integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx$ is equal to

(a) $\frac{\pi}{4}\left(e^4 - 1\right)$

(b) $\frac{\pi}{2}\left(e^2 - 1\right)$

(c) $\frac{\pi}{4}\left(e^4 + 1\right)$

(d) $\frac{\pi e^2}{4}$

(e) $\frac{\pi e^{16}}{4}$

22. If volume of a tetrahedron formed by the plane $ax + y - z = 4$ and the three coordinate planes is $\frac{16}{3}$, then value of a is

(a) -2

(b) 2

(c) -3

(d) 4

(e) 0

23. The volume of the solid enclosed by the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$ is

(a) $\frac{16}{3}$

(b) 8

(c) $\frac{2}{3}$

(d) 4

(e) 1

24. The value of $\iiint_E \sqrt{x^2 + y^2 + z^2}\, dV$, where E is the solid that lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ and above the xy-plane, is

(a) $\frac{65\pi}{2}$

(b) $\frac{65\pi}{4}$

(c) $\frac{56\pi}{4}$

(d) $\frac{65\pi^2}{4}$

(e) $\frac{65\pi}{8}$

25. The triple integral that gives the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cone $z^2 = x^2 + y^2$ is

(a) $\displaystyle\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_0^{\sqrt{2}} \rho^2 \sin\phi \, d\rho d\phi d\theta$

(b) $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} \rho^2 \sin\phi \, d\rho d\phi d\theta$

(c) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\pi} \int_{-\sqrt{2}}^{\sqrt{2}} \rho^2 \sin\phi \, d\rho d\theta d\phi$

(d) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \rho^2 \sin\phi \, d\phi d\theta d\rho$

(e) $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\frac{\pi}{2}}^{\frac{7\pi}{4}} \rho^2 \sin\phi \, d\phi d\rho d\theta$