1. (a) (2 points) Verify that \( e^y = y - x^2 + C \) is an implicit solution of the differential equation
\[
\frac{dy}{dx} = \frac{2x}{1 - e^y}.
\]

(b) (2 points) Use the implicit solution given in part (a) to solve the initial-value problem (IVP)
\[
\frac{dy}{dx} = \frac{2x}{1 - e^y}, \quad y(1) = 1.
\]

(c) (2 points) Tell whether the IVP given in part (b) has a unique solution. Justify your answer.
2. (7 points) Solve: \( \sin x \frac{dy}{dx} - y \cos x = \sin^2 x \), where \( 0 < x < \frac{\pi}{2} \).
3. (7 points) Solve: \( e^{x^3+y^2} \, dx + \frac{y}{x^2} \, dy = 0. \)
4. (7 points) Solve: \((ye^x + \sin x)dx + (2y + e^x + \cos y)dy = 0\).
5. (6 points)

(a) Use an appropriate substitution to reduce the following differential equation
\[
\frac{dy}{dx} = \frac{2y - x}{x + 3y}
\]
to a separable equation.

(b) Is it possible to write the separable equation obtained in (a) as a linear differential equation? Justify your answer.
6. (7 points) According to Newton’s Law of cooling/warming, the rate of change of temperature \( T(t) \) of an object at any time \( t \) is proportional to the difference between \( T \) and the surrounding temperature \( T_m \). Let \( k \) be the constant of proportionality.

(a) Write the differential equation that models this phenomenon.

(b) Solve the differential equation found in (a) and write its general solution as \( T(t) = T_m + ce^{kt} \).

(c) An object of temperature 10°C is left in a room of temperature 30°C. After 2 minutes the object temperature is 15°C. How long will it take for the object to reach 25°C?