Solution **Quiz 2**

1. Here $P = e^{2x}\sin 2y, Q = e^{2x}\cos 2y$

$$\begin{aligned}\frac{\partial Q}{\partial x} &= 2e^{2x}\cos 2y \\ \frac{\partial P}{\partial y} &= 2e^{2x}\cos 2y.\end{aligned}$$

So $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0.$ [3 marks]

Note: For this P and Q, the integral $\int_C P\,dx + Q\,dy$ would be zero for any closed curve C because the functions P and Q are defined everywhere.

2.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = \vec{i}\left(\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right) - \vec{j}\left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right)$$
$$+ \vec{k}\left(\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial y}(yz)\right) = 0.$$

So the integral is independent of the path joining P and Q.

Find ϕ so that $\frac{\partial \varphi}{\partial x} = yz, \quad \frac{\partial \varphi}{\partial y} = xz,$

$$\frac{\partial \varphi}{\partial z} = xy.$$

From first equation, $\varphi = xyz + g(y,z)$. From second equation, $\frac{\partial \varphi}{\partial y} = xz + \frac{\partial g}{\partial y} = xz$. So $\frac{\partial g}{\partial y} = 0$. Hence g depends only on z, say $g(y,z) = h(z)$. So $\varphi = xyz + h(z)$ and from the third equation $\frac{\partial \varphi}{\partial z} = xy + h'(z) = xy$. So $h'(z) = 0$. Thus $h(z)$ is a constant. We can take this constant to be zero.
So $\varphi(x,y,z) = xyz$

$$\begin{aligned}\int_P^Q yzdx + xzdy + xydz &= \phi(Q) - \varphi(P) \\ &= 64 - 1 = 63.\end{aligned}$$

[7 marks]

Note: You want to find φ sos that

$$\begin{aligned}d_\varphi &= \frac{\partial \varphi}{\partial x}dx + \frac{\partial \varphi}{\partial y} + \frac{\partial \varphi}{\partial z}dz \\ &= yzdx + xzdy + xydz.\end{aligned}$$

So it is easy to guess the answer: $\varphi(x,y,z) = xyz$.