1. Write clearly.

2. Show all your steps.

3. No credit will be given to wrong steps.

4. Do not do messy work.

5. Calculators and mobile phones are NOT allowed in this exam.
1. (4 marks) Suppose that $V$ and $W$ are two subspaces of $\mathbb{R}^n$. Let

$$K = V \cap W = \{k \mid k \in V \text{ and } k \in W\}.$$ 

Prove that $K$ is a subspace of $\mathbb{R}^n$.

2. (2 marks) Let $A, B$ and $C$ be $n \times n$ real matrices. Suppose $BA = AC = I_n$. Prove that $B = C$. 
3. (8 marks) Suppose that $S$ consists of all vectors in $\mathbb{R}^4$ with zero second component.

(a) Show that $S$ is a subspace of $\mathbb{R}^4$.

(b) Determine a basis for $S$.

(c) Determine the dimension of the basis.
4. (8 marks) Given the system

\[
\begin{align*}
    x_1 + 3x_2 - a^2 x_3 &= a^2 \\
    2x_1 + 3x_2 + ax_3 &= 2 \\
    3x_1 + 4x_2 + 2x_3 &= 3.
\end{align*}
\]

Find a condition on \( a \) such that the non-homogenous system has a unique solution, no solution and infinitely solution.
5. (10 marks) Given a matrix \( A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \)

(a) Show that \( A \) is diagonalizable.

(b) Find the eigenvalues of \( A^{100} \).
6. (8 marks) Find value(s) for $a, b$ and $c$ for which the following matrix will be orthogonal

$$A = \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{3} & a \\ -\frac{3}{\sqrt{5}} & -\frac{1}{3} & b \\ -\frac{3}{\sqrt{5}} & \frac{1}{3} & c \end{bmatrix}$$
7. (5 marks) Given \( x = 2t^2, \quad y = 3t^2, \quad z = 4t^2 \) and \( 1 \leq t \leq 4 \). Find the following

(a) Write the position vector of the curve and tangent vector for the curve whose parametric equations are given.

(b) Find a length function \( s(t) \) for the curve.

(c) Write the position vector as a function of \( s \).

(d) Verify that the resulting position vector has a derivative of length 1.
8. (5 marks) Show that the following matrix is singular.

\[ B = \begin{bmatrix}
3 & 3 & 6 \\
0 & 1 & 2 \\
-2 & 0 & 0 \\
\end{bmatrix} \]