

**King Fahd University of Petroleum and Minerals**  
**Department of Mathematics and Statistics**  
**Math 411 Major Exam I**  
**First Semester 2008–2009(081)**

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Name: \_\_\_\_\_

ID #: \_\_\_\_\_

Sec#: \_\_\_\_\_

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1. Suppose  $x = (x_1, x_2, \dots, x_n, \dots)$ ,  $y = (y_1, y_2, \dots, y_n, \dots)$  are infinite sequences such that  $\sum_{i=1}^{\infty} x_i^2$ ,  $\sum_{i=1}^{\infty} y_i^2$  both converge. Prove that the series  $\sum_{i=1}^{\infty} x_i y_i$  converges. [10 pts]
2. (a) Use the intermediate value theorem to prove that for any continuous function  $f : [0, 1] \rightarrow [0, 1]$ , there is a number  $x$  so that  $f(x) = x$ . [7 pts]  
(b) If  $f, g$  are one to one and onto functions on a closed interval  $[0, 1]$ , show that  $f(x) = g(x)$  for some  $x$  in  $[0, 1]$ . [3 pts]
3. Prove that the complement of the disc  $D = \{(x, y) : x^2 + y^2 \leq 1\}$  is a connected set. [10 pts]
4. (a) Find critical points of  $f(x, y, z) = xy + yz$ . [5 pts]  
(b) Write  $f$  as a sum or difference of squares. [5 pts]