1. [6pts] Let \( d = (423, 198) \). Find \( d \) and find integers \( x \) and \( y \) such that \( 423x + 198y = d \).
2. [6pts] (a) Show that if $a, b$ are positive integers then $(a^2, b^2) = (a, b)^2$.
(b) Is it true that $[a^2, b^2] = [a, b]^2$? Either prove this or give a counterexample.
3. [6pts] (a) Show that the number of primes is infinite.
(b) Show that if \( p \) is a prime number and \( a, b \in \mathbb{N} \) are such that \([a, b] = p(a, b)\), then either \( a | b \) or \( b | a \).
4. [6pts] (a) State without proof Wilson’s theorem, and use it to prove that if \( p \) is prime then \( p \mid (p - 2)! - 1 \).
(b) Show that for each integer \( n \), 23 does not divide \( 4n^2 + 4 \).
5. [6pts] Let $a, b, m, r$ be positive integers such that $a \equiv b \pmod{m}$. Show that $a^r \equiv b^r \pmod{m}$. Is it true that $a^r \equiv b^r \pmod{mr}$? Either prove or give a counterexample.