1. [6pts] (a) Prove that if $p$ is prime and $d|(p - 1)$ then $x^d \equiv 1 \pmod{p}$ has $d$ solutions.
(b) How many solutions does $x^2 \equiv 1 \pmod{30}$ have? Justify.
2. [6pts] (a) Solve the system: $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, $x \equiv 5 \pmod{6}$, $x \equiv 1 \pmod{7}$
(b) Find the number of solutions of $48x \equiv 87 \pmod{105}$, and then solve this congruence.
3. [9pts] (a) Let $N = 2^\alpha p^\beta$, where $p$ is an odd prime and $\alpha, \beta$ are positive integers. Find $p$ if $5\phi(N) = 2N$.
(b) Show that if the positive integer $r$ satisfies $2\phi(r) = r$, then $r$ must be a power of 2.
4. [9pts] (a) Show that if \( a \) has order \( h \pmod{m} \) and if \( a^k \equiv 1 \pmod{m} \), then \( h \mid k \).
(b) Prove that if \( g \) is a primitive root \( \pmod{17} \), then \( 17 - g \) is also a primitive root \( \pmod{17} \).
(c) How many solutions does \( x^2 \equiv 13 \pmod{17} \) have? Same question for: \( x^{20} \equiv 6 \pmod{37} \).