Math 132 03/07/2009

Quiz 1

Ex1. Find the following limits.

a) $\lim_{x \to 4} \dfrac{x^2 - 9x + 20}{x^2 - 3x - 4}$; b) $\lim_{x \to +\infty} \dfrac{3 - 4x - 2x^3}{5x^3 - 8x + 1}$

Ex2. Consider the function

$$f(x) = \begin{cases} \dfrac{x^2 + x - 2}{x + 2} & \text{if } x > -2 \\ x + 5 & \text{if } x \leqslant -2 \end{cases}$$

a) Does $\lim_{x \to -2} f(x)$ exist? Justify your answer.

b) Is f continuous at -2? Justify your answer.

Solutions

Ex1 a) $\lim_{x \to 4} \dfrac{x^2 - 9x + 20}{x^2 - 3x - 4} = \lim_{x \to 4} \dfrac{(x-4)(x-5)}{(x-4)(x+1)} = \lim_{x \to 4} \dfrac{x-5}{x+1}$

$= \dfrac{\lim_{x \to 4} x - 5}{\lim_{x \to 4} x + 1} = \dfrac{4-5}{4+1} = -\dfrac{1}{5}$

b) $\lim_{x \to +\infty} \dfrac{3 - 4x - 2x^3}{5x^3 - 8x + 1} = \lim_{x \to +\infty} \dfrac{-2x^3}{5x^3} = \lim_{x \to +\infty} \dfrac{-2}{5} = -\dfrac{2}{5}$

Ex2 a) $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} x + 5 = -2 + 5 = +3$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \dfrac{x^2 + x - 2}{x + 2} = \lim_{x \to -2^+} \dfrac{(x-1)(x+2)}{x+2}$

$= \lim_{x \to -2^+} x - 1 = -2 - 1 = -3$

Since $\lim_{x \to -2^+} f(x) \neq \lim_{x \to -2^-} f(x)$, then $\lim_{x \to -2} f(x)$ does not exis

b) Since $\lim_{x \to -2} f(x)$ does not exist, then f connot be continuous at -2.

<u>Conclusion</u>: f is discontinuous at -2.