Math 132 — Quiz 4 — 03/28/200

Ex1: Find $\frac{dy}{dx}$ in the following cases

a) $y = 4^{\ln x} \log_2(x^2+1)$

b) $xe^y + \ln(xy) = xy$

c) $y = \left(\frac{\ln x}{x}\right)^x$

Ex2. Consider the curve $e^{x+y} = 1 + \ln(1+x+y)$.

a) Find the slope of the tangent line to the curve at (0,0)
b) Find an equation of the tangent line to the curve at (0,0)
c) Find the y-intercept of the tangent line.

Solutions

Ex1: a) Since $\frac{d}{dx}4^{\ln x} = \frac{\ln 4}{x}4^{\ln x}$

and $\frac{d}{dx}\log_2(x^2+1) = \frac{2x}{(x^2+1)\ln 2}$, we have:

$$\frac{dy}{dx} = \frac{\ln 4}{x}4^{\ln x}\log_2(x^2+1) + 4^{\ln x}\frac{2x}{(x^2+1)\ln 2}$$

$$= 4^{\ln x}\left[\frac{\ln 4}{x}\log_2(x^2+1) + \frac{2x}{(x^2+1)\ln 2}\right]$$

b) $\frac{d}{dx}\left(xe^y + \ln(xy)\right) = \frac{d}{dx}(xy)$

$$\Rightarrow e^y + xe^y\frac{dy}{dx} + \frac{y + x\frac{dy}{dx}}{xy} = y + x\frac{dy}{dx}$$

$$\Rightarrow e^y + xe^y \frac{dy}{dx} + \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = y + x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(xe^y + \frac{1}{y} - x\right) = y - \frac{1}{x} - e^y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - \frac{1}{x} - e^y}{xe^y + \frac{1}{y} - x} = \frac{y}{x}\,\frac{xy - 1 - xe^y}{xye^y + 1 - xy}$$

$$= \frac{y}{x}\,\frac{\ln(xy) - 1}{xye^y + 1 - xy}$$

c) $\ln y = x\ln\left(\frac{\ln x}{x}\right) = x\left[\ln(\ln x) - \ln x\right]$

$$\Rightarrow \frac{dy}{dx}\cdot\frac{1}{y} = \left[\ln(\ln x) - \ln x\right] + x\left[\frac{1}{x\ln x} - \frac{1}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{\ln x}{x}\right)^x\left(\underbrace{\ln(\ln x) - \ln x}_{\ln\left(\frac{\ln x}{x}\right)} + \frac{1}{\ln x} - 1\right)$$

Ex2: a) $\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}\left[1 + \ln(1 + x + y)\right]$

$$\Rightarrow \left(1 + \frac{dy}{dx}\right)e^{x+y} = \frac{1 + \frac{dy}{dx}}{1 + x + y}$$

$$\Rightarrow \left(1 + \frac{dy}{dx}\right)e^{x+y}(1 + x + y) = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left[e^{x+y}(1 + x + y) - 1\right] = 1 - e^{x+y}(1 + x + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - e^{x+y}(1 + x + y)}{e^{x+y}(1 + x + y) - 1} = -1$$

Hence the slope of the tangent line at $(0,0)$ is -1

b) An equation of the tangent line at $(0,0)$ is:

$$y = -x$$

c) This tangent line intercepts the y-axis at the point $(0,0)$.