(1) Show that the function \( u(x, y) = e^{-x}cosy - e^{-y}cosx \) is a solution of Laplace’s equation.

(2) Calculate \( f_{xy} \) if \( f(x, y) = 3xy^4 + x^3y^2 \).

(3) Find the tangent plane to the surface \( z = 9x^2 + y^2 + 6x + 5 \) at the point \((1, 2, 8)\).

(4) Find the linear approximation of the function \( f(x, y) = \sqrt{x^2 + y^2} \) at the point \((3, 4)\) and use it to approximate \( \sqrt{(2.9^2) + (4.1^2)} \).

(5) Let \( z = xln(2x + 3y), x = sint \) and \( y = cost \). Find \( \frac{dz}{dt} \) at \( t = \frac{\pi}{2} \). [Ans. \(-\frac{3}{2}\)]

(6)(a) Find the rate of change of \( f(x, y) = y ln x \) at point \((1, -3)\) in the direction of the vector \( \vec{u} = \langle \frac{4}{5}, \frac{3}{5} \rangle \).

\( (b) \) Find the equations of the tangent plane and normal line to the surface \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \) at the point \((x_0, y_0, z_0)\).

(7) Find the local maximum, local minimum and saddle points of the function \( f(x, y) = x^4 + y^4 - 4xy + \sqrt{5} \).

(8) Find the shortest distance from the point \((1, 0, -2)\) to the plane \( x + 2y + z = 4 \).

(9) Find the absolute maximum and minimum values of the function \( f(x, y) = x^2 = y^2 + x^2y + 4 \) on the set \( D = \{(x, y) : \ mod\ x \leq 1, \ mod\ y \leq 1\} \).

(10) Find the points on the sphere \( x^2 + y^2 + z^2 = 36 \) that are closest to and farthest from the point \((1, 2, 2)\). [ANS. \((2, 4, 4), (-2, -4, -4)\)]

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April 17, 2009