(1) Show that a group $G$ is abelian if and only if the map $f : G \rightarrow G$ given by $f(x) = x^{-1}$ is an automorphism.

(2) Let $G = < a >$ be a cyclic group and let $H$ be any subgroup. Show that every homomorphism $f : G \rightarrow H$ is completely determined by $f(a)$.

(3) Let $G$ be a finite group of order $2n$. Show that $G$ contains an element of order 2.
(4) Let $G$ be a group and let $H, K$ be subgroups of $G$.
(a) Show by example that the set $HK$ need not to be a subgroup of $G$.
(b) Show that $HK$ is a subgroup of $G$ if $G$ is abelian.

(5) Show that $Aut(Z_p) \cong Z_{p-1}$ ($p$ prime).
(6) Construct the subgroup lattice of \( Z_{50} \).
(7) T/F. If true prove it otherwise give a counter example.

(a) Let $G$ be a finite cyclic group. If $n || |G|$, then $\exists$ a subgroup of order $n$.

(b) Let $G$ be a group. If $a^2 = e$ for all $a \in G$, then $G$ is abelian.

(c) Every left coset is a right coset for some subgroup.