Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Total mark

Table of Marks

King Fahd University of Petroleum and Minerals

First Exam for Math 513

Semester 2, Academic year 2008-2009

Time allowed 1 hour and 55 minutes

Full Name: .....

ID Number: .....

Note: Show all your work and write clear steps

Laplace Transform

Question 1 Find the Laplace transform of $f(t) = \int_0^t \tau \mathrm{H}(\tau - 1)\, d\tau$.

Question 2 Find the Laplace inverse of the functions:

a) $F(s) = \frac{s+1}{s^2-4s}$

b) $F(s) = \frac{1}{s^2(s+1)^2}$

Question 3 Use Laplace transform to solve the following system:

$$\begin{cases} x' + 2x - y' - y = 0 \\ -x' + y + 2\int_0^t x(q)\mathrm{e}^{-(t-q)}\, dq = \mathrm{e}^{-t} \end{cases}$$

with $x(0) = y(0) = 0$.

Fourier Transform

Question 4 Use the contour integration technique to find the inverse Fourier transform of

$$F(s) = \frac{1}{s^2 - 2is - 5}\,.$$

Question 5 Use Fourier transform to solve the DE:

$$y'' - 4y = \mathrm{e}^{-3t}\mathrm{H}(t).$$

Fourier Series

Question 6

a) Plot the graph of the function $f(t) = \begin{cases} 1 & -2 \le t < -1 \\ -t & -1 \le t < 0 \\ t & 0 \le t < 1 \\ 1 & 1 \le t < 2\,. \end{cases}$

b) Expand $f(t)$ using an appropriate cosine or sine Fourier series.

Question 7

a) Find the complex Fourier series of $f(t) = \begin{cases} -1 & -2 \le t < 0 \\ 1 & 0 \le t < 2\,. \end{cases}$

b) Find the frequency spectrum of the periodic extension of f.