Q 1	Q 2	Q 3	Q 4	Q 5	Total mark

Table of Marks

King Fahd University of Petroleum and Minerals

Second Exam for Math 513

Semester 2, Academic year 2008-2009

Time allowed: Two hours

Full Name: ....

ID Number: ....

Note: Show all your work and write clear steps

Question 1 Use the method of separation of variables to solve the BVP:

$$\begin{cases} u_t = u_{xx}, & -\pi < x < \pi, \quad t > 0 \\ u(\pi, t) - u(-\pi, t) = 0, & t > 0 \\ u_x(\pi, t) - u_x(-\pi, t) = 0, & t > 0 \\ u(x, 0) = f(x), & -\pi < x < \pi. \end{cases}$$

Question 2 Solve the BVP:

$$\begin{cases} u_{xx} = u_{tt}, \quad 0 < x < 3, \quad t > 0, \\ u(0, t) = 0, \quad u(3, t) = 0, \quad t > 0, \\ u(x, 0) = f(x), \quad u_t(x, 0) = 0, \quad 0 < x < 3, \end{cases}$$

where

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1, & 1 \le x \le 2 \\ 3 - x, & 2 \le x \le 3. \end{cases}$$

Question 3 Use an appropriate Fourier transform method to solve the BVP:

$$\begin{cases} u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, \quad y > 0 \\ u_y(x, 0) = 0, \quad 0 < x < \pi \\ u(0, y) = 0, \quad u(\pi, y) = f(y), \quad y > 0. \end{cases}$$

Question 4 Consider the BVP:

$$\begin{cases} u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0 \\ u(0, t) = u(L, t) = 0, \quad t > 0 \\ u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad 0 < x < L. \end{cases}$$

By the Fourier series theorem and the odd periodic extension trick, $u(x, t)$ has, for each fixed t, a unique expansion

$$u(x, t) = \sum_{n=1}^{\infty} b_n(t) \sin(n\pi x/L).$$

a- Substitute this expansion in the given wave equation and show that

$$b_n''(t) + \left(\frac{cn\pi}{L}\right)^2 b_n(t) = 0.$$

b- Start from the given expansion of u to solve the given BVP.

Question 5 Consider the following BVP:

$$\begin{cases} u_t = u_{xx}, \quad 0 < x < 1, \quad t > 0 \\ u(0,t) = T_1, \quad u(1,t) = T_2, \quad t > 0 \\ u(x,0) = f(x), \quad 0 < x < 1. \end{cases}$$

Let $\lim_{t \longrightarrow \infty} u(x,t) = u_E(x)$ where $u_E(x)$ is the equilibrium temperature. Note, $u_E(x)$ satisfies the given the heat equation and boundary conditions.

a- Substitute $u(x,t) = v(x,t) + u_E(x)$ in the given BVP and then solve the obtained BVP for v.

b- Use part (a) to solve the given BVP for u.