

Q 1	Q 2	Q 3	Q 4	Total mark

Table of Marks

# King Fahd University of Petroleum and Minerals

## Third Exam for Math 513

Semester 2, Academic year 2008-2009

**Time allowed: 90 minutes**

Full Name: .....

ID Number: .....

**Note:** Show all your work and write clear steps

**Question 1** Consider the following Sturm-Liouville problem:

$$\begin{cases} y'' + (\lambda - 4)y = 0, & 0 < x < 1, \\ y'(0) + 2y(0) = 0, \\ y'(1) + 2y(1) = 0. \end{cases}$$

- Find the eigenvalues and the corresponding eigenfunctions.
- Verify that at least one of the obtained eigenfunctions is not an orthonormal eigenfunction.

**Question 2** Let  $A$  be any square matrix and  $A^T$  be its transpose

- If  $A^T = 2A$ , show that  $A$  is a zero matrix
- If  $A$  is a  $2 \times 2$  invertible matrix with  $A^3 = 2A$ . Show that  $|A| = 2$  or  $-2$ .
- Use the characteristic equation of  $A$  to show that  $A$  and  $A^T$  have the same eigenvalues.

**Question 3** Solve the following system of DEs:

$$\begin{cases} x' = 3x \\ y' = x + 3y + z \\ z' = -x + 2z. \end{cases}$$

**Hint:** One of the eigenvalues is repeated, however, you should be able to find two linearly independent solutions corresponding to it directly.

**Question 4** The Sturm-Liouville problem:

$$\begin{cases} y'' + (\lambda - 4)y = 0, & 0 < x < 1, \\ y'(0) + 2y(0) = 0, \\ y'(1) + 2y(1) = 0 \end{cases}$$

has eigenfunctions  $y_0(x) = e^{-2x}$  and  $y_n(x) = 2\sin(n\pi x) - n\pi\cos(n\pi x)$  for  $n \geq 1$ . Using this to find the eigenfunctions expansion for  $f(x) = e^{2x}$ .