

MATH 574

Assignment 1 Due Date: 23 of May, 2009

Consider the following two-point BVP:

$$-4u'' - u = 1 \quad \text{for } 0 < x < \pi \quad \text{with } u(0) = u(\pi) = 0.$$

Here u is a function of x .

a) Verify that $u = \cos(x/2) + \sin(x/2) - 1$ is a solution of the given BVP.

b) Use a uniform partition with \mathbf{M} subintervals each of length \mathbf{h} to define the piecewise linear FEM of the given BVP.

c) Write the proposed scheme (in part (b)) in a matrix form.

d) Choose $\mathbf{M} = 20$. Compute the finite element solution at the nodes of the partition. Use the obtained datum to plot the approximate solution. Plot the exact solution on the same figure and compare between them.

e) Say $u_{\mathbf{h}}$ is your approximate solution. Compute the error

$$\max_{0 \leq i \leq \mathbf{M}} |u(z_i) - u_{\mathbf{h}}(z_i)| \quad \text{for } \mathbf{M} = 20, 40 \text{ and } 80$$

where $z_i = \frac{i}{2}\mathbf{h}$ for $i = 1, 3, 5, \dots, (2\mathbf{M} - 1)$. Find the order of convergence.