MATH 574

Assignment 3 Due Date: 17th of June, 2009

Problems 1

Consider the following BVP:

$$\begin{cases} -\Delta u = f, \quad 0 < x < 1, \quad 0 < y < 1 \\ u_y(x,0) = u_y(x,1) = 0, \quad 0 < x < 1 \\ u(0,y) = u(1,y) = 0, \quad 0 < y < 1. \end{cases}$$

Here u and f are functions of x and y.

a) Define the piecewise bilinear FEM of the given BVP over a quasi-uniform triangular mesh. (All details are required)

b) Write the proposed scheme in a matrix form.

c) Is it possible to use a rectangular mesh instead of triangular one? If yes, in which finite dimensional space the FE solution lies?

Problem 2

Consider the following 2-point BVP (subject to homogeneous Neumann boundary condition):

$$\begin{cases} -u_{xx} + (x - 8\pi^2)u = x\sin^2(2\pi x) - 8\pi^2\cos^2(2\pi x), \quad 0 < x < 1, \\ u_x(0) = u_x(1) = 0. \end{cases}$$

Here u and f are functions of x. Given that the exact solution $u = \sin^2(2\pi x)$.

a) Define the piecewise linear FEM of the given 2-point BVP using a uniform mesh consisting of $\mathbf{M}$ subintervals each of length h.

b) Write the proposed scheme in a matrix form.

c) Say that $u_{\mathbf{h}}$ is your approximate solution. Use the L_2-norm to compute error between u and $u_{\mathbf{h}}$ for $\mathbf{M} = 20$, 40 and 80. Find the order of convergence.

d) Plot u and $u_{\mathbf{h}}$ (for $\mathbf{M} = 20$) on the same figure and compare between them.

Problems 3

Consider the following parabolic problem:

$$\begin{cases} u_t - u_{xx} = f(x) \quad \text{for } 0 < x < 1 \text{ and } 0 < t < 1 \\ u(0,t) = u(1,t) = 0 \quad \text{for } 0 < t < 1 \\ u(x,0) = g(x) \quad \text{for } 0 < x < 1. \end{cases}$$

Study the piecewise linear FEM in both time and space for solving the given problem.

Hint Use a uniform mesh in both time and space, define your finite dimensional space, define the scheme and finally write it in a matrix form.