1. Consider the non-linear ODE

\[ x^3 y'' = y, \quad x = 0 \text{ irregular singular point} \]

Assuming solution \( y = e^{s(x)} \)

Show that \( s(x) \sim 2x^{-1/2} + \frac{3}{4} \ln x + D(x) \)

Determine \( D(x) \), for \( x \to 0^+ \).

What is leading behavior of \( y(x) \) as \( x \to 0^+ \)?

2. Consider \( y'' = y^2 + x \), \( x \to \infty \)

Show that \( y(x) \) has spontaneous singularities by putting \( y = \sqrt{x} \) \( u(x) \) and a suitable transformation in independent variable obtain an estimate in the form of an elliptic integral.

3. Obtain asymptotic approximations for solution of the following for large \( Z \)

\[ W'' + \frac{1}{Z} F(Z) W = 0 \]

Where \( F(Z), \sim a_1 + \frac{a_2}{Z}, \ldots, \quad Z \to \infty \)

4. Discuss asymptotic solutions of \( W'' - Z^2 W = 0, \quad Z \to \infty \) in the form \( W(Z) \sim e^{\phi_0(z) + \phi_1(z) + \ldots} \), \( Z \to \infty, \{\phi_n(Z)\} \) being an asymptotic sequence for large \( Z \).

5. Consider \( W''(\lambda + x)W = 0, \quad \lambda \to \infty \)

\( W(0, \lambda) = a \)
\( W'(0, \lambda) = b \), \( x \geq 0 \)

\( a, b \) constants and \( x = O(1) \).

Assuming \( W(x, \lambda) \sim \exp \left( g_0(\lambda)\psi_0(x) + g_1(\lambda)\psi_1(x) \right) \)

Determine the \( WKB \) solution for \( x \to \infty \).