Solution

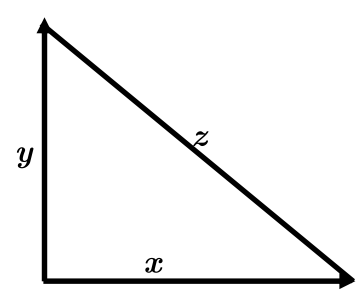
Answer the following questions.

Question 1: Write the formula for $f^{(n)}(x)$, where $f(x) = \dfrac{1}{1-2x}$.

Solution:

$$
\begin{aligned}
f'(x) &= \frac{2}{(1-2x)^2} \\
f''(x) &= \frac{1\cdot 2\cdot 2^2}{(1-2x)^3} \\
f'''(x) &= \frac{1\cdot 2\cdot 3\cdot 2^3}{(1-2x)^4} \\
f^{(4)}(x) &= \frac{1\cdot 2\cdot 3\cdot 4\cdot 2^4}{(1-2x)^5} \\
&\vdots \\
&\vdots \\
f^{(n)}(x) &= \frac{n!2^n}{(1-2x)^{n+1}}
\end{aligned}
$$

Question 2: Two cars start moving from the same point. One travels North at $\frac{3}{2}$ mi/h and the other travels east at 2 mi/h . At what rate is the distance between the cars increasing tow hours later?



Solution: According to the adjacent figure,

We have: $\dfrac{dx}{dt} = 2 \text{ mi/h}$ and $\dfrac{dy}{dt} = \dfrac{3}{2} \text{ mi/h}$

After two hours $x = 4 \text{ mi}$ and $y = 3 \text{ mi}$. We want $\dfrac{dz}{dt}$.

Now we have $z^2 = x^2 + y^2$. So

$$z^2 = 4^2 + 3^2 = 25 \Rightarrow z = 5 \text{ mi}$$

and hence,

$$
\begin{aligned}
2z\frac{dz}{dt} &= 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \\
z\frac{dz}{dt} &= x\frac{dx}{dt} + y\frac{dy}{dt} \\
5\frac{dz}{dt} &= 4\cdot 2 + 3\cdot\frac{3}{2} = \frac{25}{2}
\end{aligned}
$$

Therefore, the distance between the cars increases at a rate of $5/2 \text{ mi/h}$.

Solution

Question 3: Using the linearization find the approximate value of $\sin(0.1)$.

Solution: Let $f(x) = \sin(x), \quad a = 0$. Then $f'(x) = \cos(x), f(0) = 0$ and $f'(0) = 1$. So

$$f(x) = \sin(x) \approx f(0) + f'(0)x = x \qquad \text{(near } 0\text{)}$$

Therefore,

$$\sin(0.1) = f(0.1) \approx 0.1$$

Question 4: Find $f'(\ln 2)$, if $f(x) = \coth^{-1}(\cosh x)$.

Solution: $f'(x) = \dfrac{\sinh x}{1 - \cosh^2 x} = \dfrac{\sinh x}{-\sinh^2 x} = -\csc hx$. Now we find

$$f'(\ln 2) = -\csc \mathrm{h}(\ln 2) = \frac{-2}{2 - \frac{1}{2}} = \frac{-2}{\frac{3}{2}} = \frac{-4}{3}$$

Question 5: If $f(x) = x^{3+x^3}$, find $f'(1)$.

Solution: $\ln f(x) = \left(3 + x^3\right)\ln x$, then

$$\frac{f'(x)}{f(x)} = \frac{3 + x^3}{x} + 3x^2 \ln x$$

$$f'(x) = x^{3+x^3}\left(\frac{3 + x^3}{x} + 3x^2 \ln x\right)$$

Therefore, $f'(1) = 1^{3+1^3}\left(\dfrac{3 + 1^3}{1} + 3{\cdot}1^2 \ln 1\right) = 4$