KFUPM　　Summer Term (083)　Name ________ serial ____

MATH 102-3-4　Quiz#3 (I & II)　ID# KEY　section ____

Part I ① (4-points) Determine whether the series $\sum_{n=1}^{\infty} \frac{2^n + \cos n}{5^n}$ is convergent or divergent.

$$\frac{2^n + \cos n}{5^n} < \frac{2^n + 1}{5^n} = \left(\frac{2}{5}\right)^n + \left(\frac{1}{5}\right)^n \quad 1\,pt$$

Now $\sum \left(\frac{2}{5}\right)^n$ and $\sum \left(\frac{1}{5}\right)^n$ are two convergent geometric series　1 pt

$\Rightarrow \sum \left[\left(\frac{3}{5}\right)^n + \left(\frac{1}{7}\right)^n\right]$ is convergent　1 pt

$\Rightarrow$ the given series is convergent by the comparison test　1 pt

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② (4-points) Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln(5n)}{n}$ is convergent or divergent.

Let $b_n = \frac{\ln 5n}{n} \Rightarrow \lim_{n\to\infty} b_n$ is of $\frac{\infty}{\infty}$ type　1 pt

But $\lim_{x\to\infty} \frac{\ln 5x}{x} \overset{H.R}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0 \Rightarrow$

$\lim_{n\to\infty} b_n = 0$ (i)　1 pt

Also $f(x) = \frac{\ln 5x}{x} \Rightarrow f'(x) = \frac{1 - \ln 5x}{x} < 0$ for all $x \geq 1$

$\Rightarrow b_n$ is decreasing (ii)　1 pt

(i) & (ii) $\Rightarrow$ The series is convergent by the alternating series Test.

Part II $\rightarrow$

$\boxed{\text{Part II}}$ ① (3-points) How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ do we need to add to ensure that its sum is accurate to within 0.01

$|\text{error}| \le b_{n+1} < 0.01$ 1pt

$\frac{1}{(n+1)^2} < \frac{1}{10^2} \Rightarrow n+1 > 10 \Rightarrow n > 9$ 1pt

Thus we need at least 10 terms 1pt

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② (5-points) Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4+1}$ is absolutely convergent, conditionally convergent, or divergent

$|a_n| = \frac{n^3}{n^4+1}$, $b_n = \frac{1}{n}$

$\Rightarrow \lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim_{n\to\infty} \frac{n^4}{n^4+1} = 1 > 0$ 1pt

But $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent $\Rightarrow \sum_{n=2}^{\infty} \frac{n^3}{n^4+1}$ is dgt.

by the limit comparism test. 1pt

Now $b_n = \frac{n^3}{n^4+1} \longrightarrow 0$ as $n \to \infty$ 1pt

and $\frac{d}{dx}\left(\frac{x^3}{x^4+1}\right) = \frac{3x^2(x^4+1) - 4x^6}{(x^4+1)^2} = \frac{-x^6+3x^2}{(x^4+1)^2} < 0, \ x \ge 2$

$\Rightarrow$ decreasing 1pt $\Rightarrow$ the series is conditionally convergent 1pt

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MATH 102-3-4 Quiz #3 (I&II) ID# KEY section ____

Part I ① (4-Points) Determine whether the series

$\sum_{n=1}^{\infty} \left(\frac{3^n + \sin n}{7^n} \right)$ is convergent or divergent.

$$\frac{3^n + \sin n}{7^n} < \frac{3^n + 1}{7^n} = \left(\frac{3}{7}\right)^n + \left(\frac{1}{7}\right)^n \quad \text{1 pt}$$

Now $\sum \left(\frac{3}{7}\right)^n$ and $\sum \left(\frac{1}{7}\right)^n$ are two convergent geometric (1 pt)

series $\Rightarrow \sum \left[\left(\frac{3}{7}\right)^n + \left(\frac{1}{7}\right)^n\right]$ is convergent (1 pt)

$\Rightarrow$ the given series is convergent by the comparison test (1 pt)

② (4-Points) Determine whether the alternating series

$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln(7n)}{n}$ is convergent or divergent.

Let $b_n = \frac{\ln 7n}{n} \Rightarrow \lim_{n \to \infty} b_n$ is of $\frac{\infty}{\infty}$ type (1 pt)

But $\lim_{x \to \infty} \frac{\ln 7x}{x} \overset{H.R}{=} \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = 0 \Rightarrow$ (1 pt)

$\lim_{n \to \infty} b_n = 0$ (i)

Also $f(x) = \frac{\ln 7x}{x} \Rightarrow f'(x) = \frac{1 - \ln 7x}{x^2} < 0$ for all $x \geq 1$

$\Rightarrow b_n$ is decreasing (ii) (1 pt)

(i) & (ii) $\Rightarrow$ The series is convergent by the alternating series test. (1 pt)

Part II $\rightarrow$

• $\boxed{\text{Part II}}$ ① (3-points) How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4}$ do we need to add to ensure that its sum is accurate to within 0.0001

$|\text{error}| \leq b_{n+1} < 0.0001$ (1 pt)

$\frac{1}{(n+1)^4} < \frac{1}{10^4} \Rightarrow n+1 > 10 \Rightarrow n > 9$ (pt)

Thus we need at least 10 terms. (1 pt)

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② (5-points) Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n n^2}{n^3+1}$ is absolutely convergent, conditionally convergent, or divergent.

$|a_n| = \frac{n^2}{n^3+1}$, $b_n = \frac{1}{n}$

$\Rightarrow \lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim_{n\to\infty} \frac{n^3}{n^3+1} = 1 > 0$ (1 pt)

But $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent $\Rightarrow \sum \frac{n^2}{n^3+1}$ is divergent by the limit comparison test. (1 pt)

Now $b_n = \frac{n^2}{n^3+1} \longrightarrow 0$ as $n \longrightarrow \infty$ (1 pt)

and $\frac{d}{dx}\left(\frac{x^2}{x^3+1}\right) = \frac{2x(x^3+1) - 3x^4}{(x^3+1)^2} = \frac{-x^4+2x}{(x^3+1)^2} < 0, x \geq 2$

$\Rightarrow$ decreasing (1 pt) $\Rightarrow$ the series is conditionally convergent (1 pt)