Math 132 Quiz 3 07/25/2009

Find $\frac{dy}{dx}$.

(a) $y = x^{\frac{1}{x}}$, (b) $y = (\ln x)^x$, (c) $y = \sqrt{\frac{(x+3)(x-2)}{2x-1}}$

Solutions

(a) $y = x^{\frac{1}{x}} = e^{\frac{1}{x}\ln x} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x}\ln x\right) \cdot e^{\frac{1}{x}\ln x}$

$= \left(-\frac{1}{x^2}\ln x + \frac{1}{x^2}\right) e^{\frac{1}{x}\ln x} = \frac{1}{x^2}(1-\ln x)\, x^{\frac{1}{x}} = x^{\frac{1}{x}-2}(1-\ln x)$

$= x^{\frac{1-2x}{x}}(1-\ln x).$

2nd Method:

$$\ln y = \ln\left(x^{\frac{1}{x}}\right) = \frac{1}{x}\ln x$$

$\Rightarrow \frac{d}{dx}\ln y = \frac{d}{dx}\left[\frac{1}{x}\ln x\right]$. So that:

$$\frac{\frac{dy}{dx}}{y} = -\frac{\ln x}{x^2} + \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = \frac{y}{x^2}(1-\ln x) = x^{\frac{1-2x}{x}}(1-\ln x).$$

(b) $y = (\ln x)^x = e^{x\ln(\ln x)} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[x\ln(\ln x)\right] \cdot e^{x\ln(\ln x)}$

$= \left(\ln(\ln x) + x\frac{\frac{1}{x}}{\ln x}\right) e^{x\ln(\ln x)} = \left(\ln(\ln x) + \frac{1}{\ln x}\right)(\ln x)^x$

2nd Method $\ln y = \ln\left[(\ln x)^x\right] = x\ln(\ln x)$. So

$\frac{d}{dx}\ln y = \frac{d}{dx}\left[\ln\left((\ln x)^x\right)\right] = \frac{d}{dx}\left[x\ln(\ln x)\right]$. Thus

$$\frac{\frac{dy}{dx}}{y} = \ln(\ln x) + x\frac{\frac{1}{x}}{\ln x} \Rightarrow \frac{dy}{dx} = \left[\ln(\ln x) + \frac{1}{\ln x}\right] y$$

Hence $\frac{dy}{dx} = \left[\ln(\ln x) + \frac{1}{\ln x}\right](\ln x)^x$.

(c) $\ln y = \ln\sqrt{\dfrac{(x+3)(x-2)}{2x-1}} = \dfrac{1}{2}\ln\dfrac{(x+3)(x-2)}{2x-1}$

$= \dfrac{1}{2}\left[\ln[(x+3)(x-2)] - \ln(2x-1)\right]$

$= \dfrac{1}{2}\left[\ln(x+3) + \ln(x-2) - \ln(2x-1)\right]$

After differentiating both sides, we obtain:

$$\frac{\frac{dy}{dx}}{y} = \frac{1}{2}\left[\frac{1}{x+3} + \frac{1}{x-2} - \frac{2}{2x-1}\right]$$

<u>Hence</u>: $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{1}{x+3} + \dfrac{1}{x-2} - \dfrac{2}{2x-1}\right)\ln\sqrt{\dfrac{(x+3)(x-2)}{2x-1}}$